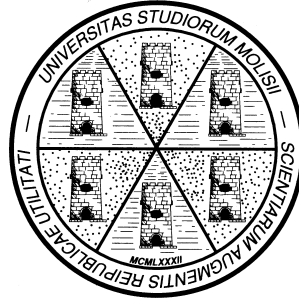


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Change in persistence tests for panels: an update and some new results

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Change in persistence tests for panels: an update and some new results.*

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Abstract

In this paper we propose a set of new panel tests to detect changes in persistence. The test statistics are used to test the null hypothesis of stationarity against the alternative of a change in persistence from $I(0)$ to $I(1)$, from $I(1)$ to $I(0)$ and in an unknown direction. The limiting distributions of the panel tests are derived, and small sample properties are investigated by Monte Carlo experiments under the hypothesis that the individual series are independently cross-section distributed. These tests have a good size and power properties. Cross-sectional dependence is also considered. A procedure of de-factorizing, proposed by Stock and Watson (2002), is applied. The defactored panel tests have good size and power. The empirical results obtained from applying these tests to a panel covering 21 OECD countries observed between 1970 and 2007 suggest that inflation rate changes from $I(1)$ to $I(0)$ when cross-correlation is considered.

Keywords: Persistence, Stationarity, Panel data.

JEL Classification: C12, C23.

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1 Introduction

Recent time series literature has shown that economic and financial data are characterized by a change in persistence between separate $I(1)$ and $I(0)$ regimes rather than simply $I(1)$ or $I(0)$ behavior. For example, Cogley and Sargent (2001) and Emery (1994), using post World War II data, argued that persistence in U.S. inflation has decreased substantially since the early 1980s. Strikingly, Emery (1994) finds that U.S. inflation in the 1980s can best be described as a white noise. Further evidence of change in persistence from $I(1)$ to $I(0)$ behavior in U.S. inflation is also reported in Kim (2000), Buseti and Taylor (2004) and Leybourne et al. (2003). Other variables for which changes in persistence have been observed include real output (e.g Taylor, 2005) and short-term interest rates (e.g. Mankiw *et al.*, 1987)).

A number of testing procedures have been developed to test for changes in persistence. The most popular of these are the ratio-based change in persistence tests of Kim (2000), Kim *et al.* (2002), Buseti and Taylor (2004) and Harvey *et al.* (2006). These test the null hypothesis that a series is a constant $I(0)$ process against the alternative that it displays a change in persistence, either from $I(0)$ to $I(1)$, or viceversa. Kim (2000) and Kim *et al.* (2002) proposed a residual-based ratio test against changes in persistence in a time series, focusing on the case of a shift from $I(0)$ to $I(1)$ at some point in the sample. Kim (2000) also discussed the possibility of $I(1)$ to $I(0)$ shifts but did not provide tests against such an alternative. Buseti and Taylor (2004) proposed new ratio-based tests and breakpoint estimators which are consistent under $I(1)$ to $I(0)$ changes, and they demonstrated that the ratio-based tests which are consistent against changes from $I(1)$ to $I(0)$ are not consistent against changes from $I(0)$ to $I(1)$, and viceversa, with neither consistent against constant $I(1)$ processes. Harvey *et al.* (2006) developed a set of new tests which are based on a modified version of the ratio-base statistics of Kim (2000), Kim *et al.* (2002) and Buseti and Taylor (2004). These modifications use the variable addition approach of Vogelsang (1998), and a recent generalization by Sayginsoy (2003), and yielding tests which, by design, have the same critical values regardless of whether the process is $I(0)$ or (near) $I(1)$ throughout. This technique can only be used with the ratio based test of the null $I(0)$ because other tests of the $I(0)$ ($I(1)$) null are based on statistics which are divergent under constant $I(1)$ ($I(0)$) processes. Hence the null hypothesis is constant persistence (either a constant $I(0)$ process or a constant $I(1)$ process), and the alternative is a change in persistence. Using a panel framework, Costantini and Gutierrez (2007) proposed new recursive ADF unit root tests to detect changes in persistence based on the inverse normal Z test suggested by Choi (2001). The small sample properties of the recursive tests are investigated by Monte Carlo experiments. The panel tests have good size and power.

In this paper we propose a set of new panel tests to detect changes in persistence. The test statistics are used to test the null hypothesis of stationarity against the alternative of a change in persistence from $I(0)$ to $I(1)$, from $I(1)$ to $I(0)$ and an unknown direction. Two set of panel tests are proposed. The first set is based on the hypothesis that the individual series are independently

cross-section distributed. The second one uses the hypothesis of cross-sectional dependence.

The paper is organized as follows. In section 2 we present a set of new panel tests to detect a change in persistence under the hypothesis of cross-section independence. Section 3 describes the panel tests under the cross-section dependence hypothesis. Section 4 presents Monte Carlo simulations. In section 5 we apply the tests to analyze a panel of 21 OECD inflation rate series for the period 1970.1-2007.3 . Section 6 concludes. The main technical proofs and derivations are in the Appendix.

2 Persistence tests without cross-section correlation

2.1 The model

Consider the following Gaussian unobserved components model for a sample of N cross-sections observed over T time periods:

$$y_{i,t} = d_{i,t} + \mu_{i,t} + \varepsilon_{i,t}, \quad i = 1, \dots, N, t = 1, \dots, T, \quad (1)$$

we allow for the following three cases:

- Case 1: $I(0) \rightarrow I(1)$

$$\mu_{i,t} = \mu_{i,t-1} + 1(t > [T\tau])\eta_{i,t}, \quad i = 1, \dots, N, t = 1, \dots, T, \quad (2)$$

- Case 2: $I(1) \rightarrow I(0)$

$$\mu_{i,t} = \mu_{i,t-1} + 1(t \leq [\tau T])\eta_{i,t} \quad i = 1, \dots, N, t = 1, \dots, T. \quad (3)$$

- Case 3: unknown direction $I(0) \rightarrow I(1)$ or $I(1) \rightarrow I(0)$

where $1(\cdot)$ is the indicator function, $d_{i,t}$ is a deterministic component, $\varepsilon_{i,t}$ and $\eta_{i,t}$ are mutually independent mean zero iid gaussian processes with $\sigma_{\varepsilon_i}^2$ and $\sigma_{\eta_i}^2$ variance. The deterministic components are taken to be the unity vector.

From (2), it can be easily seen that for each cross section i , the data generating process yields a process which is stationary up to and including time $[\tau T]$, with the change-point proportion $\tau \in (0, 1)$, but is $I(1)$ after the break, if and only if, $\sigma_{\eta_i}^2 > 0$. Otherwise, from (3), we note that for each cross section i , the data generating process yields a process which is $I(1)$ up to and including time $[\tau T]$ but it is stationary after the break, if and only if, $\sigma_{\eta_i}^2 > 0$.

Therefore, the panel test of stationarity against a shift in persistence from stationarity to a unit root or viceversa can be framed in testing the null hypothesis as:

$$\mathbf{H}_0 = \sigma_{\eta_i}^2 = 0, \quad \forall i \quad (4)$$

against the alternative hypothesis

$$\mathbf{H}_1 = \sigma_{\eta_i}^2 > 0, \text{ at least for some } i. \quad (5)$$

When $I(0) \rightarrow I(1)$, we denote the alternative hypothesis as \mathbf{H}_{01} . If $I(1) \rightarrow I(0)$, then we use \mathbf{H}_{10} .

The following assumption plays a key role in the rest of the paper.

Assumption 1 *The process $\{\mu_{i,t}\}_{t=0}^{+\infty}$ is such that for each i*

1. $\mathbf{E}[\mu_i] = 0$;
2. $\mathbf{E}|\mu_i|^4 < +\infty$;
3. *fixed i , then $\{\mu_{i,t}\}_{t=0}^{+\infty}$ is ϕ -mixing with mixing coefficients $\phi_{i,m}$ such that*

$$\sum_{m=1}^{\infty} \phi_{i,m}^{\gamma_i} < +\infty,$$

for some $\gamma_i > 0$;

4. *There exists the long-run variance*

$$\sigma_{\mu i}^2 = \sum_{j=0}^{\infty} \mathbf{E}[\mu_{i,j+1} \mu'_{i,1}];$$

5. *for each $s \in (0, 1)$, we have*

$$\lim_{T \rightarrow \infty} \mathbf{V} \left[\frac{1}{\sqrt{T}} \sum_{t=1}^{[sT]} \mu_{i,t} \right] = s \sigma_{\mu i}^2$$

and

$$\lim_{T \rightarrow \infty} \mathbf{V} \left[\frac{1}{\sqrt{T}} \sum_{t=[sT]+1}^T \mu_{i,t} \right] = (1-s) \sigma_{\mu i}^2$$

The above conditions have been used by Phillips (1987), Phillips and Perron (1988) and Phillips and Solo (1992), among others, to prove results on the asymptotic distribution of a stochastic process. Finally, note that throughout the next sections we use sequential limits, where $T \rightarrow \infty$ is followed by $N \rightarrow \infty$.

2.2 Panel ratio-based tests: $I(0) \rightarrow I(1)$

In this section we present new panel tests to detect changes in persistence as in (2) and investigate their asymptotic behavior. We show that panel tests are standard normally distributed.

Consider the gaussian process (1)-(2). We want to test the null hypothesis \mathbf{H}_0 in (4) against \mathbf{H}_1 in (5). Let $\tilde{\varepsilon}_{i,t}$, $i = 1, \dots, N$ and $t = 1, \dots, T$, be the residuals from the regression of $y_{i,t}$ on intercept. If a structural change occurs at time $t = [\tau T]$ for $\tau \in (0, 1)$, the following partial sum process can be defined as:

$$\begin{cases} S_{i,t}^{(0)} = \sum_{j=1}^t \tilde{\varepsilon}_{i,j} & t = 1, \dots, [T\tau]; \quad i = 1, \dots, N, \\ S_{i,t}^{(1)} = \sum_{j=[T\tau]+1}^t \tilde{\varepsilon}_{i,j} & t = [T\tau] + 1, \dots, T; \quad i = 1, \dots, N, \end{cases} \quad (6)$$

Then, we consider the following test statistic:

$$\mathcal{K}_{T,N}(\tau) = \frac{\sqrt{N}}{\sigma} \cdot \left[\frac{(T - [T\tau])^{-2}}{[T\tau]^{-2}} \cdot \frac{1}{N} \cdot \sum_{i=1}^N \frac{\sum_{t=[T\tau]+1}^T S_{i,t}^{(1)}(\tau)^2}{\sum_{t=1}^{[T\tau]} S_{i,t}^{(0)}(\tau)^2} - \mu \right], \quad (7)$$

where

$$\mu = \frac{(T - [T\tau])^{-2} \sum_{t=[T\tau]+1}^T \mathbf{E}[S_{i,t}^{(1)}(\tau)^2]}{[T\tau]^{-2} \sum_{t=1}^{[T\tau]} \mathbf{E}[S_{i,t}^{(0)}(\tau)^2]} \quad (8)$$

and

$$\sigma = \sqrt{\frac{(T - [T\tau])^{-4} \sum_{t=[T\tau]+1}^T \mathbf{V}[S_{i,t}^{(1)}(\tau)^2]}{[T\tau]^{-4} \sum_{t=1}^{[T\tau]} \mathbf{V}[S_{i,t}^{(0)}(\tau)^2]}}. \quad (9)$$

Theorem 2.1 *Suppose that Assumption 1 is true under the null hypothesis \mathbf{H}_0 . Then*

$$\lim_{N \rightarrow +\infty} \lim_{T \rightarrow +\infty} \mathcal{K}_{T,N}(\tau) = \mathcal{K}(\tau) \sim N(0, 1). \quad (10)$$

In (7), the true value of τ is unknown. If the true change period is unknown three transformations of the tests $\mathcal{K}_{T,N}(\tau)$ defined in (7) for testing changes in persistence with unknown break point $[T\tau]$ can be considered:

- A maximum-Chow-type test, as used in Davies (1977), Hawkins (1987), Kim and Siegmund (1989), and Andrews (1993) is

$$\mathbf{H}_1(\mathcal{K}_{T,N}(\tau)) := \sup_{\tau \in (0,1)} \mathcal{K}_{T,N}(\tau). \quad (11)$$

- The mean score test proposed by Hansen (1991)

$$\mathbf{H}_2(\mathcal{K}_{T,N}(\tau)) := \int_{\tau \in (0,1)} \mathcal{K}_{T,N}(\tau) d\tau. \quad (12)$$

- The mean-exponential test introduced by Andrews and Ploberger (1994), i.e.

$$\mathbf{H}_3(\mathcal{K}_{T,N}(\tau)) := \log \left\{ \int_{\tau \in (0,1)} \exp[\mathcal{K}_{T,N}(\tau)] d\tau \right\}. \quad (13)$$

The asymptotic distribution of the tests defined in (11), (12) and (13) are given in the next result.

Theorem 2.2 *The following conditions hold true.*

(i) *It results*

$$\lim_{T \rightarrow +\infty} \lim_{N \rightarrow +\infty} \mathbf{H}_j(\mathcal{K}_{T,N}(\tau)) = \mathbf{H}_j(\mathcal{K}(\tau)), \quad j = 1, 2, 3.$$

(ii) *For each $j = 1, 2, 3$, we have $\mathbf{H}_j(\mathcal{K}(\tau)) \sim N(0, 1)$.*

2.3 Panel reverse test: $\mathbf{I}(1) \rightarrow \mathbf{I}(0)$

Consider the gaussian process (1)-(3). In this case, the null hypothesis refers to the stationary process and the alternative to a shift from $\mathbf{I}(1)$ to $\mathbf{I}(0)$. The following reverse test statistic is proposed:

$$\mathcal{K}_{T,N}^*(\tau) = \frac{\sqrt{N}}{\sigma} \cdot \left[\frac{[T\tau]^{-2}}{(T - [T\tau])^{-2}} \cdot \frac{1}{N} \cdot \sum_{i=1}^N \frac{\sum_{t=1}^{[T\tau]} S_{i,t}^{(0)}(\tau)^2}{\sum_{t=[T\tau]+1}^T S_{i,t}^{(1)}(\tau)^2} - \mu \right], \quad (14)$$

where

$$\mu = \mathbf{E} \left[\frac{[T\tau]^{-2} \sum_{t=1}^{[T\tau]} S_{i,t}^{(0)}(\tau)^2}{(T - [T\tau])^{-2} \sum_{t=[T\tau]+1}^T S_{i,t}^{(1)}(\tau)^2} \right]$$

and

$$\sigma = \sqrt{\mathbf{V} \left[\frac{[T\tau]^{-2} \sum_{t=1}^{[T\tau]} S_{i,t}^{(0)}(\tau)^2}{(T - [T\tau])^{-2} \sum_{t=[T\tau]+1}^T S_{i,t}^{(1)}(\tau)^2} \right]}.$$

The asymptotic distribution of the statistics defined in (14) is shown by the following result.

Theorem 2.3 *Suppose that Assumption 1 is true under the null hypothesis \mathbf{H}_0 . Then*

$$\lim_{N \rightarrow +\infty} \lim_{T \rightarrow +\infty} \mathcal{K}_{T,N}^*(\tau) = \mathcal{K}^*(\tau) \sim N(0, 1). \quad (15)$$

In the next result we give the asymptotic distributions of the transformations H_1 , H_2 and H_3 of the test \mathcal{K}^* .

Theorem 2.4 *The following propositions hold.*

(i) *It results*

$$\lim_{T \rightarrow +\infty} \lim_{N \rightarrow +\infty} H_j(\mathcal{K}_{T,N}^*(\hat{\tau})) = H_j(\mathcal{K}^*(\tau)), \quad j = 1, 2, 3.$$

(ii) *For each $j = 1, 2, 3$, we have $H_j(\mathcal{K}^*(\tau)) \sim N(0, 1)$.*

2.4 Panel tests with unknown direction

We now discuss the case of unknown direction of changes in persistence. Three panel tests are developed and their asymptotic distributions are derived. The tests are:

$$\mathcal{M}_{T,N}^{j,*} = \frac{\sqrt{N}}{\sigma_j^*} \cdot \frac{1}{N} \cdot \sum_{i=1}^N [\max\{H_j(\tilde{\mathcal{K}}_{T,i}), H_j(\tilde{\mathcal{K}}_{T,i}^*)\} - \mu_j^*], \quad j = 1, 2, 3; \quad (16)$$

where

$$\tilde{\mathcal{K}}_{T,i} = \frac{(T - [T\tau])^{-2} \sum_{t=[T\tau]+1}^T S_{i,t}^{(1)}(\tau)^2}{[T\tau]^{-2} \sum_{t=1}^{[T\tau]} S_{i,t}^{(0)}(\tau)^2},$$

$$\tilde{\mathcal{K}}_{T,i}^* = (\tilde{\mathcal{K}}_{T,i})^{-1},$$

$$\begin{aligned}\mu_j^* &= \mathbf{E} \left[\max\{H_j(\tilde{\mathcal{K}}_{T,i}), H_j(\tilde{\mathcal{K}}_{T,i}^*)\} \right] \quad j = 1, 2, 3; \quad i = 1, \dots, N, \\ \sigma_j^* &= \sqrt{\mathbf{V} \left[\max\{H_j(\tilde{\mathcal{K}}_{T,i}), H_j(\tilde{\mathcal{K}}_{T,i}^*)\} \right]} \quad j = 1, 2, 3; \quad i = 1, \dots, N.\end{aligned}$$

The asymptotic distributions of these tests are now derived.

Theorem 2.5 *It results*

$$\lim_{N \rightarrow +\infty} \lim_{T \rightarrow +\infty} \mathcal{M}_{T,N}^{j,*} \sim N(0, 1).$$

2.5 Modified panel tests

In this section we propose panel tests that are based on the modified version of the test statistics developed in subsections 2.2-2.4. These tests have the same critical value in the limit as the corresponding unmodified tests under the null hypothesis \mathbf{H}_0 , and the same limiting critical value is also appropriate under the alternative hypothesis \mathbf{H}_1 . The modification proposed has no asymptotic effect under the null \mathbf{H}_0 , so that the limiting distribution of the modified tests is the same of the corresponding unmodified tests. The modified panel tests developed are:

$$\text{MH}_j^d(\mathcal{K}_{T,N}(\tau)) := \exp(-bJ_{1,N,T}) \cdot H_j(\mathcal{K}_{T,N}(\tau)), \quad j = 1, 2, 3; \quad (17)$$

$$\text{MH}_j^d(\mathcal{K}_{T,N}^*(\tau)) := \exp(-bJ_{1,N,T}) \cdot H_j(\mathcal{K}_{T,N}^*(\tau)), \quad j = 1, 2, 3; \quad (18)$$

$$\text{MM}_{T,N}^{j,*} := \exp(-bJ_{1,N,T}) \cdot \mathcal{M}_{T,N}^{j,*} \quad j = 1, 2, 3; \quad (19)$$

where b is a finite constant and $J_{1,N,T}$ is the arithmetic mean on N of the truncated sequences of T^{-1} times the Wald statistic $J_{1,T}^{(i)}$ for testing the joint hypothesis $\varsigma_{i,k+1} = \dots = \varsigma_{i,9} = 0$ in panel regression

$$y_{i,t} = \varepsilon_{i,t} + \sum_{j=k+1}^9 \varsigma_{i,j} t^j + \text{error}, \quad t = 1, \dots, [\tau T]; \quad i = 1, \dots, N. \quad (20)$$

Under the null hypothesis, Harvey *et al.* (2006) show that

$$\lim_{T \rightarrow +\infty} J_{1,T}^{(i)} = 1, \quad \forall i = 1, \dots, N.$$

Therefore, since we assumed independence and identical distribution with respect to the cross-sectional dimension i , we have

$$\lim_{T,N \rightarrow +\infty} J_{1,N,T} = \lim_{T,N \rightarrow +\infty} \frac{1}{N} \sum_{i=1}^N J_{1,T}^{(i)} = 1.$$

Consequently, modified panel tests have the same limiting distribution under \mathbf{H}_0 as the unmodified tests. Under the alternative hypothesis, using Harvey *et al.*'s (2006) results and the fact that the asymptotic distributions of tests $H_j(\mathcal{K}_{T,N}(\tau))$, $H_j(\mathcal{K}_{T,N}^*(\tau))$, and $\mathcal{M}_{T,N}^{j,*}$, $j = 1, 2, 3$ are standard gaussian (see Theorem 2.2, Theorem 2.4 and Theorem 2.5), for each $j = 1, 2, 3$, we have

$$\lim_{N \rightarrow +\infty} T^{-2}(\text{MH}_j^d(\mathcal{K}_{T,N}(\tau)) - H_j(\mathcal{K}_{T,N}(\tau))) =$$

$$\begin{aligned}
&= T^{-2} \lim_{N \rightarrow +\infty} (\text{MH}_j^d(\mathcal{K}_{T,N}(\tau)) - \text{H}_j(\mathcal{K}_{T,N}(\tau))) = \\
&= T^{-2} \lim_{N \rightarrow +\infty} \{\exp[-bJ_{1,N,T}] - 1\} \text{H}_j(\mathcal{K}_{T,N}(\tau)) = O_p(1)O_p(1) = O_p(1).
\end{aligned}$$

Analogously, it results

$$\lim_{N \rightarrow +\infty} T^{-2} (\text{MH}_j^d(\mathcal{K}_{T,N}^*(\tau)) - \text{H}_j(\mathcal{K}_{T,N}^*(\tau))) = O_p(1)$$

and

$$\lim_{N \rightarrow +\infty} T^{-2} (\text{MM}_{T,N}^{j*} - \mathcal{M}_{T,N}^{j*}) = O_p(1).$$

Thus, we obtain a test which rejects for large values of the modified tests and retains the same rate of consistency under the alternative \mathbf{H}_{01} as the original unmodified H_j tests. For the alternative \mathbf{H}_{10} , the modified tests are also $O_p(1)$. A more appropriate modification procedure is proposed for testing the null against the alternative \mathbf{H}_{01} . Following Harvey *et al.* (2006), we modify tests (17), (18) and (19) by introducing

$$J_{N,\min} := \min_{\tau \in (0,1)} J_{1,N,[\tau T]}.$$

We define

$$\text{MH}_{j,\min}(\mathcal{K}_{T,N}(\tau)) := \exp(-bJ_{N,\min}) \cdot \text{H}_j(\mathcal{K}_{T,N}(\tau)), \quad j = 1, 2, 3; \quad (21)$$

$$\text{MH}_{j,\min}(\mathcal{K}_{T,N}^*(\tau)) := \exp(-bJ_{N,\min}) \cdot \text{H}_j(\mathcal{K}_{T,N}^*(\tau)), \quad j = 1, 2, 3; \quad (22)$$

$$\text{MM}_{\min,T,N}^{j*} := \exp(-bJ_{N,\min}) \cdot \mathcal{M}_{T,N}^{j*} \quad j = 1, 2, 3; \quad (23)$$

and

$$J_{\min} := \lim_{N \rightarrow +\infty} J_{N,\min}.$$

in this case also we can derive the asymptotic analysis using the alternative hypothesis of Harvey *et al.* (2006). We have

$$\begin{aligned}
&\lim_{N \rightarrow +\infty} T^{-2} (\text{MH}_{j,\min}(\mathcal{K}_{T,N}(\tau)) - \text{H}_j(\mathcal{K}_{T,N}(\tau))) = \\
&= T^{-2} \lim_{N \rightarrow +\infty} (\text{MH}_{j,\min}(\mathcal{K}_{T,N}(\tau)) - \text{H}_j(\mathcal{K}_{T,N}(\tau))) = \\
&= T^{-2} \lim_{N \rightarrow +\infty} \{\exp[-bJ_{N,\min}] - 1\} \text{H}_j(\mathcal{K}_{T,N}(\tau)) = \\
&= T^{-2} \{\exp[-bJ_{\min}] - 1\} \lim_{N \rightarrow +\infty} \text{H}_j(\mathcal{K}_{T,N}(\tau)) = o_p(1)O_p(1) = o_p(1),
\end{aligned}$$

and, analogously,

$$\lim_{N \rightarrow +\infty} T^{-2} (\text{MH}_{j,\min}(\mathcal{K}_{T,N}^*(\tau)) - \text{H}_j(\mathcal{K}_{T,N}^*(\tau))) = o_p(1)$$

and

$$\lim_{N \rightarrow +\infty} T^{-2} (\text{MM}_{\min,T,N}^{j*} - \mathcal{M}_{T,N}^{j*}) = o_p(1).$$

Thus, the new modification has no asymptotic effect under the alternative \mathbf{H}_{01} , unlike to the original MH_j^d modifications. Under the alternative \mathbf{H}_{10} , it is easily shown that the modified min tests are $O_p(1)$.

2.6 Estimation of the break

In this section we present a procedure to estimate the unknown change point. Consider the following estimator:

$$\Pi_{N,T}(\tau) = \frac{1}{N} \sum_{i=1}^N \frac{\sum_{t=[T\tau]+1}^T \tilde{\mu}_{i,t}^2 / (T - [T\tau])^2}{\sum_{t=1}^{[T\tau]} \tilde{\mu}_{i,t}^2 / [\tau T]}. \quad (24)$$

In order to explore the asymptotic behavior of the estimated unknown change point, the following assumption is necessary.

Assumption 2 *Let $\tilde{\mu}_{i,s+1}, \tilde{\mu}_{i,s+2}, \dots, \tilde{\mu}_{i,s+m}$, for $s \in 0, \dots, T-1$ and $m \leq T-s$ be a sequence of stationary variables. Assume that $m^{-1} \sum_{t=s+1}^{s+m} \tilde{\mu}_{i,t}^2 \rightarrow E[\mu_i^2]$ for $E[\mu_i^2] < \infty$, $\forall i = 1, \dots, N$.*

Now, let $\hat{\tau}$ be:

$$\hat{\tau} = \left\{ \operatorname{argmax}_{\tau \in (0,1)} \Pi_{N,T}(\tau) \right\}. \quad (25)$$

The following theorem shows the asymptotic properties of $\hat{\tau}$:

Theorem 2.6 *Suppose that Assumption 2 holds. Under the alternative hypothesis, it results*

$$(\hat{\tau} - \tau) = o_p(1), \quad (26)$$

$$T(\hat{\tau} - \tau) = O_p(1), \quad (27)$$

3 Persistence test with cross-section correlation

Previous testing procedures are valid under the assumption that the units are cross-section independent. However, this requirement is rarely likely to be satisfied in empirical economic applications where the countries or regions depend on each other. Thus in this section we therefore generalize our earlier results to suit the case of dependence among the cross-sectional units, by filtering out common factor in the panel structure. Basically, the estimation procedure is based on the Stock and Watson (2002), Bai (2003) and Bai and Ng (2004) approaches. It consists of two steps. In the first step, the common factors are estimated using the Principal Component (PC) method. In order to estimate the true number of common factors accurately in this first step, we compute the number of factors using Bai and Ng's (2002) selection criteria. In the second step, defactored data is constructed.

Specifically, let, as before, $y_{i,t}$ be the observation on the i -th cross section unit at time t for $i = 1, \dots, N$, $t = 1, \dots, T$ and suppose that it is generated according to the following linear heterogeneous panel data model:

$$y_{i,t} = d_{i,t} + \mu_{i,t} + F_t \lambda_i + \varepsilon_{i,t} \quad (28)$$

As in the previous section, three cases are considered:

- Case 1: $I(0) \rightarrow I(1)$

$$\mu_{i,t} = \mu_{i,t-1} + 1(t > [\tau T])\eta_{i,t} \quad (29)$$

- Case 2: $I(1) \rightarrow I(0)$

$$\mu_{i,t} = \mu_{i,t-1} + 1(t \leq [\tau T])\eta_{i,t} \quad (30)$$

- Case 3: unknown direction $I(0) \rightarrow I(1)$ or $I(1) \rightarrow I(0)$

where, as before, $\varepsilon_{i,t}$ is a stationary process, $\tau \in (0, 1)$ and the deterministic component $d_{i,t}$ is taken to be the unity vector. Further we assume that F_t in (28) is a stationary $(r \times 1)$ vector of common factors and λ_i is the corresponding vector of factor loadings.

Let us write the factors F_t and factor loadings λ_i in (28) in matrix notation. We write $F = (F_1, \dots, F_T)'$ and $\Lambda = (\lambda_1, \dots, \lambda_N)'$. The following assumptions are required:

Assumption 3 *the loadings λ_i is either deterministic such that $\|\lambda_i\|^4 \leq M$ or stochastic such that $E\|\lambda_i\|^4 \leq M$, in either case $\Lambda_i'\Lambda_i/N \rightarrow \Sigma_\Lambda$, as $N \rightarrow \infty$ for some $(r \times r)$ positive definite matrix Σ_Λ .*

Assumption 4 *$E\|F_t\|^4 \leq M$ and $\frac{1}{T} \sum_{t=1}^T F_t F_t' \rightarrow \Sigma_F$, for a $(r \times r)$ positive definite matrix Σ_F .*

Assumption 5 *The errors $\{\varepsilon_{i,t}\}$ and $\{\eta_{i,t}\}$, the factor $\{F_t\}$ and the loadings $\{\lambda_i\}$ are four mutually independent stochastic variables.*

Assumption 3 and 4 imply r common factors and assumption 5 is standard in factor analysis. The common factors are estimated as they are in Stock and Watson (2002), i.e. using principal components. Specifically, the principal component of F , denoted as \tilde{F} , is \sqrt{T} times the first r eigenvectors, corresponding to the first r largest eigenvalues of the $(T \times T)$ matrix of demeaned and standardized $\tilde{y}_i \tilde{y}_i'$. Under the normalization $\tilde{F} \tilde{F}'/T = I_r$, the estimated loading matrix is $\tilde{\Lambda} = \tilde{F}' \tilde{y}_i/T$. Thus the estimated residuals are defined as

$$\tilde{z}_{i,t} = \tilde{y}_{i,t} - \tilde{F}_t \tilde{\lambda}_i \quad (31)$$

From the data generating process (28)-(29) and (31), one can, for example, see that for each cross section i , the process $\tilde{z}_{i,t}$ is stationary up to and including time $[\tau T]$ but is $I(1)$ after the break if, and only if, $\sigma_{\eta_i}^2 > 0$. Naturally the converse it's true if we adopt (30) instead of (29).

Thus our strategy is to apply the panel test statistics presented in section 2 to the de-factored data $\tilde{z}_{i,t}$.

4 Monte Carlo simulation results

In this section we use Monte Carlo experiments to examine the finite sample properties of the panel persistence tests. We consider two sets of Monte Carlo experiments. The first set focuses on the model (1)-(3), i.e where we assume cross-section independence, while the second set of experiments is based on the model (28)-(31) where we allow for the presence of dependence across the different units in the panel. We start the analysis considering the empirical

rejection frequencies of the tests when the data are generated according to the $I(0)$ - $I(1)$ switch data generating process embraced in (1)-(3) under the hypothesis of cross-section independence. We investigate the impact of varying the signal-noise-ratio among $\sigma_{\eta i} = 0, 0.10, 0.50$ and $\sigma_{\varepsilon i} \sim U[0.5, 1.5]$ and the breakpoint among $\tau = 0.3, 0.5, 0.7$. The simulation results were performed using 1000 Monte Carlo replications and the RNDN function of Gauss 6.0. As is often the case in the literature, we fix for all the tests $\Lambda = [0.2, 0.8]$ and $T = 50, 100$ and $N = 25, 50, 100$. In Table 1 we present the moments of Kim's (2000, 2002) and Busetti and Taylor's (2004) tests which were used to standardize the panel tests. Their values were computed using 50,000 replications.

Table 1 about here

The size results for the benchmark model (1)-(3) are reported in Table 2.

Table 2 about here

All the panel test statistics seem to have good size for both small and large T, N .

Looking at power of the tests for the case $I(0) \rightarrow I(1)$ (Tables 3-5), many interesting results emerge.

Table 3 about here

Table 4 about here

Table 5 about here

Comparing the power of panel tests derived using Kim's (2000), Busetti and Taylor's (2004) and Harvey *et al.*'s (2006) methods we find no significant differences. As expected, the power of tests grows, with the exception of reverse panel tests, as the signal to noise ratio rises and τ is smaller. This occurs because the higher σ_{η} is, the stronger is the random walk component. We have that the smaller τ is, the greater is the proportion of the sample containing a random walk component. Finally, the previous finding are reversed for the panel reverse tests. This because the fact that we are testing a change from $I(0)$ to $I(1)$. The opposite results are found for a shift from $I(1) \rightarrow I(0)$. Here, as expected, the power of tests grows largely for the reverse tests and it is striking that the results mimic those in Tables 3-5. Thus $H_1(\mathcal{K}^*)$, $H_2(\mathcal{K}^*)$ and $H_3(\mathcal{K}^*)$ show better properties than $H_1(\mathcal{K})$, $H_2(\mathcal{K})$, $H_3(\mathcal{K})$ and \mathcal{M}^{1*} , \mathcal{M}^{2*} and \mathcal{M}^{3*} (see Tables 6-8).

Table 6 about here

Table 7 about here

Table 8 about here

We now present the empirical size of tests when cross-section dependence is included in the model, as in equations (28)-(30). We consider two cases of cross section dependence, a "low cross section dependence", where $\lambda_i \sim iidU[0.0, 0.2]$, and "high cross section dependence", with $\lambda_i \sim iidU[-1, 3]$. In the simulation we include only one factor $F_t \sim N(0, 1)$. Results are reported in panels A (low case) and B (high case) of Table 9.

Table 9 about here

As expected, the extent of over-rejection of the tests very much depends on the degree of cross-section dependence. Both for low as well as for strong cross-section dependence the panel tests are distorted with over-rejection, which grows as the degree of cross-section dependence rise. Thus panel tests that do not allow for cross-section dependence can be seriously biased if the degree cross-section dependence is large. To take into account of cross-section dependence we use the method proposed in Stock and Watson (2002) and Bai and Ng (2004). The method basically consists of filtering out the individual-specific cross-sections y_{it} by the factor component computed using the principal component method. The number of factors are computed using the methodology proposed in Bai and Ng (2002). To be precise throughout the Monte Carlo simulation analyses the number of factors are computed using the $IC(3)$ criterion proposed in Bai and Ng (2002) with a maximum number of three factors. As before, we use only one factor $F_t \sim N(0, 1)$, and $\lambda \sim U[0, 1]$.

In Tables 10 and in 11-13, we present, respectively, the size and power of defactored panel tests using the Stock and Watson (2002) methodology. Looking at the results we note first that the tests have now generally good size. As expected, the power of tests grows for larger values of T and N .

Table 10 about here

Table 11 about here

Table 12 about here

Table 13 about here

Finally, results of the power of tests for the case of a change from $I(1)$ to $I(0)$ are reported in Tables 14-16. As expected, the power of tests grows large for the reverse tests.

Table 14 about here

Table 15 about here

Table 16 about here

5 Empirical applications

We apply the panel tests described in this paper to a panel of 21 OECD quarterly inflation rate series observed for the period 1970.1-2007.3.¹ The series are calculated as the first difference of the logarithm of the (seasonally adjusted) consumer price index. The data are taken from OECD Main Economic Indicators. In Table 17 the panel tests results are reported.

Table 17 about here

Looking at their values, we note first that for first set of test statistics, i.e. test statistics which are computed not taking into account possible cross-section

¹The countries included in the panel are Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Italy, Japan, Luxembourg, the Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, the United Kingdom, the United States.

dependence, reverse tests strongly suggest a change of persistence from $I(1)$ to $I(0)$. Change in persistence are also shown by the \mathcal{M}^j tests as well as their modified version. Mixed responses are obtained from the $H_1(\mathcal{K})$ test, when this is compared with $H_2(\mathcal{K})$ and $H_3(\mathcal{K})$ test statistics. While the former test rejects the null hypothesis, the latter tests do not reject the null hypothesis for a process which is $I(0)$ throughout. In order to take into account possible cross-dependence across the countries, we first compute the number of factors. The $IC(3)$ criterion suggests three factors (we allow for a maximum number of five factors). We use the estimated factors and factor loadings to compute $\hat{\tilde{z}}_{i,t}$, as in equation (31). Looking at the results of panel test statistics we note that the previous results are now partially reversed. Here both $H_1(\mathcal{K})$, and $H_2(\mathcal{K})$, $H_3(\mathcal{K})$ test statistics do not reject the null hypothesis, allowing for a process which is $I(0)$ throughout the sample. Reversed test statistics indicate a change from $I(1)$ to $I(0)$ and \mathcal{M}^j strongly reject the null hypothesis of constant persistence (constant $I(0)$). Given these results, we conclude that the inflation process is characterized by a change in persistence from $I(1)$ to $I(0)$. Using the cross-sectional dependence adjusted series, the change in persistence, computed using (25), took place in 1980.4.

6 Conclusions

In this paper we present new panel test statistics for a change in persistence which are based on a modified time series version of the ratio-based statistics presented in Busetti and Taylor (2004) and Harvey *et al.* (2006). These statistics are used to test the null hypothesis of stationarity against the alternative of a change in persistence from $I(0)$ to $I(1)$, or viceversa. The alternative of an unknown direction is also considered. Asymptotic distributions of the new panel tests under the hypothesis of cross-section independence are derived and Monte Carlo analysis suggests that these tests perform very well. Cross-section dependence is also considered.

We show first that when testing for a change in persistence from $I(0)$ ($I(1)$) to $I(1)$ ($I(0)$) panel tests have good properties. Secondly, we report the importance of taking into account possible cross-sectional dependence when computing the panel test statistics, especially for highly dependent panels. Finally, we apply the panel tests to a panel of 21 OECD inflation rates observed during the period 1970.1 - 2007.3. The results were consistent with a change of persistence from $I(1)$ to $I(0)$ in April 1980.

Appendix A. Proof of theorems

Proof of Theorem 2.1. Fixed $i = 1, \dots, N$, $t_1 = 1, \dots, [T\tau]$ and $t_2 = [T\tau] + 1, \dots, T$, it results $S_{i,t_1}^{(1)}$ and $S_{i,t_2}^{(0)}$ mutually independent. Therefore, (8) and (9) can be rewritten as

$$\mu = \mathbf{E} \left[\frac{(T - [T\tau])^{-2} \sum_{t=[T\tau]+1}^T S_{i,t}^{(1)}(\tau)^2}{[T\tau]^{-2} \sum_{t=1}^{[T\tau]} S_{i,t}^{(0)}(\tau)^2} \right]$$

and

$$\sigma = \sqrt{\mathbf{V} \left[\frac{(T - [T\tau])^{-2} \sum_{t=[T\tau]+1}^T S_{i,t}^{(1)}(\tau)^2}{[T\tau]^{-2} \sum_{t=1}^{[T\tau]} S_{i,t}^{(0)}(\tau)^2} \right]}.$$

By Theorem 3.1 in Kim (2000), it results

$$\lim_{T \rightarrow +\infty} \frac{(T - [T\tau])^{-2} \sum_{t=[T\tau]+1}^T S_{i,t}^{(1)}(\tau)^2}{[T\tau]^{-2} \sum_{t=1}^{[T\tau]} S_{i,t}^{(0)}(\tau)^2} = \frac{(1 - \tau)^{-2} \int_{\tau}^1 V_i(r - \tau)^2 dr}{\tau^{-2} \int_0^{\tau} V_i(r)^2 dr},$$

where $\{V_i\}_{i=1}^{+\infty}$ is a sequence of standard brownian bridges that are independent and identically distributed. Furthermore, by the hypotheses stated in Assumption 1, we have that

$$\begin{aligned} \lim_{T \rightarrow +\infty} \mathbf{E} \left[\frac{(T - [T\tau])^{-2} \sum_{t=[T\tau]+1}^T S_{i,t}^{(1)}(\tau)^2}{[T\tau]^{-2} \sum_{t=1}^{[T\tau]} S_{i,t}^{(0)}(\tau)^2} \right] &= \bar{\mu}, \\ \lim_{T \rightarrow +\infty} \mathbf{V} \left[\frac{(T - [T\tau])^{-2} \sum_{t=[T\tau]+1}^T S_{i,t}^{(1)}(\tau)^2}{[T\tau]^{-2} \sum_{t=1}^{[T\tau]} S_{i,t}^{(0)}(\tau)^2} \right] &= \bar{\sigma}^2, \end{aligned}$$

with

$$\bar{\mu} = \mathbf{E} \left[\frac{(1 - \tau)^{-2} \int_{\tau}^1 V_i(r - \tau)^2 dr}{\tau^{-2} \int_0^{\tau} V_i(r)^2 dr} \right] \quad (32)$$

and

$$\bar{\sigma}^2 = \mathbf{V} \left[\frac{(1 - \tau)^{-2} \int_{\tau}^1 V_i(r - \tau)^2 dr}{\tau^{-2} \int_0^{\tau} V_i(r)^2 dr} \right] \quad (33)$$

for each $i = 1, \dots, N$. Therefore

$$\lim_{N \rightarrow +\infty} \lim_{T \rightarrow +\infty} \mathcal{K}_{T,N}(\tau) = \lim_{N \rightarrow +\infty} \frac{\sqrt{N}}{\bar{\sigma}} \cdot \frac{1}{N} \cdot \sum_{i=1}^N \left[\frac{(1 - \tau)^{-2} \int_{\tau}^1 V_i(r - \tau)^2 dr}{\tau^{-2} \int_0^{\tau} V_i(r)^2 dr} - \bar{\mu} \right],$$

for $i = 1, \dots, N$. The Central Limit Theorem guarantees that

$$\lim_{N \rightarrow +\infty} \frac{\sqrt{N}}{\bar{\sigma}} \cdot \frac{1}{N} \cdot \sum_{i=1}^N \left[\frac{(1 - \tau)^{-2} \int_{\tau}^1 V_i(r - \tau)^2 dr}{\tau^{-2} \int_0^{\tau} V_i(r)^2 dr} - \bar{\mu} \right] \sim N(0, 1),$$

and the Theorem is completely proved. ■

Proof of Theorem 2.2.

- (i) The result follows from the continuous mapping theorem and the continuity of the functionals.
- (ii) Since $\mathcal{K}(\tau) \sim N(0, 1)$, for each τ , then $\mathcal{K}(\tau)$ is an iid continuous-time stochastic process. Therefore, we can define the random variable $\mathcal{K} \sim N(0, 1)$ such that $\mathcal{K}(\tau) \equiv \mathcal{K}$, for each $\tau \in (0, 1)$. Then we have

$$H_1(\mathcal{K}(\tau)) = \sup_{\tau \in (0, 1)} (\mathcal{K}) = \mathcal{K} \sim N(0, 1);$$

$$H_2(\mathcal{K}(\tau)) = \int_{\tau \in (0, 1)} \mathcal{K} d\tau = \mathcal{K} \cdot \int_{\tau \in (0, 1)} d\tau = \mathcal{K} \sim N(0, 1);$$

$$H_3(\mathcal{K}(\tau)) = \log \left\{ \int_{\tau \in (0, 1)} \exp[\mathcal{K}] d\tau \right\} = \log \{ \exp[\mathcal{K}] \} \cdot \int_{\tau \in (0, 1)} d\tau = \mathcal{K} \sim N(0, 1)$$

The result is completely proved. ■

Proof of Theorem 2.3. By Theorem 3.1 in Busetti and Taylor (2004), it results

$$\lim_{T \rightarrow +\infty} \frac{[T\tau]^{-2} \sum_{t=1}^{[T\tau]} S_{i,t}^{(0)}(\tau)^2}{(T - [T\tau])^{-2} \sum_{t=[T\tau]+1}^T S_{i,t}^{(1)}(\tau)^2} = \frac{\tau^{-2} \int_0^\tau [V_i^{***}(r)]^2 dr}{(1 - \tau)^{-2} \int_\tau^1 [V_i^{**}(r)]^2 dr},$$

where

$$V_i^{**}(r) = V_i(r) - V_i(\tau) - (r - \tau)(1 - \tau)^{-1}(V_i(1) - V_i(\tau))$$

$$V_i^{***}(r) = V_i(r) - r\tau^{-1}V_i(\tau)$$

and

$$V_i(r) = W_0(r) + c \left\{ \int_0^{\min\{r, \tau\}} W_c(s) ds + 1(r > \tau)[(r - \tau)W_c(\tau)] \right\},$$

where W is a standard Wiener process. Hypotheses stated in Assumption 1 imply

$$\lim_{T \rightarrow +\infty} \mathbf{E} \left[\frac{[T\tau]^{-2} \sum_{t=1}^{[T\tau]} S_{i,t}^{(0)}(\tau)^2}{(T - [T\tau])^{-2} \sum_{t=[T\tau]+1}^T S_{i,t}^{(1)}(\tau)^2} \right] = \bar{\mu},$$

$$\lim_{T \rightarrow +\infty} \mathbf{V} \left[\frac{[T\tau]^{-2} \sum_{t=1}^{[T\tau]} S_{i,t}^{(0)}(\tau)^2}{(T - [T\tau])^{-2} \sum_{t=[T\tau]+1}^T S_{i,t}^{(1)}(\tau)^2} \right] = \bar{\sigma}^2,$$

with

$$\bar{\mu} = \mathbf{E} \left[\frac{\tau^{-2} \int_0^\tau [V_i^{***}(r)]^2 dr}{(1 - \tau)^{-2} \int_\tau^1 [V_i^{**}(r)]^2 dr} \right] \quad (34)$$

and

$$\bar{\sigma}^2 = \mathbf{V} \left[\frac{\tau^{-2} \int_0^\tau [V_i^{***}(r)]^2 dr}{(1 - \tau)^{-2} \int_\tau^1 [V_i^{**}(r)]^2 dr} \right] \quad (35)$$

for each $i = 1, \dots, N$. Hence

$$\lim_{N \rightarrow +\infty} \lim_{T \rightarrow +\infty} \mathcal{K}_{T,N}^*(\tau) = \lim_{N \rightarrow +\infty} \frac{\sqrt{N}}{\bar{\sigma}} \cdot \frac{1}{N} \cdot \sum_{i=1}^N \left[\frac{\tau^{-2} \int_0^\tau [V_i^{***}(r)]^2 dr}{(1 - \tau)^{-2} \int_\tau^1 [V_i^{**}(r)]^2 dr} - \bar{\mu} \right],$$

for $i = 1, \dots, N$. The Central Limit Theorem guarantees that

$$\lim_{N \rightarrow +\infty} \frac{\sqrt{N}}{\bar{\sigma}} \cdot \frac{1}{N} \cdot \sum_{i=1}^N \left[\frac{\tau^{-2} \int_0^\tau [V_i^{***}(r)]^2 dr}{(1-\tau)^{-2} \int_\tau^1 [V_i^{**}(r)]^2 dr} - \bar{\mu} \right] \sim N(0, 1),$$

and the Theorem is completely proved. ■

Proof of Theorem 2.4. Analogous to the proof of Theorem 2.2. ■

Proof of Theorem 2.5. It is a direct consequence of the Central Limit Theorem. ■

Proof of Theorem 2.6. Since

$$M_T^{(i)}(\tau) := \frac{\sum_{t=[T\tau]+1}^T \tilde{\mu}_{i,t}^2 / (T - [T\tau])^2}{\sum_{t=1}^{[T\tau]} \tilde{\mu}_{i,t}^2 / [\tau T]} \geq 0, \quad \forall i = 1, \dots, N,$$

then

$$\begin{aligned} \hat{\tau} &= \operatorname{argmax}_{\tau \in (0,1)} M_T^{(i)}(\tau) \Rightarrow \\ \Rightarrow \hat{\tau} &= \operatorname{argmax}_{\tau \in (0,1)} \left\{ \frac{1}{N} \cdot \sum_{i=1}^N M_T^{(i)}(\tau) \right\} = \operatorname{argmax}_{\tau \in (0,1)} \Pi_{N,T}(\tau). \end{aligned}$$

Therefore, Theorem 3.5 in Kim (2000) guarantees the thesis. ■

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Table 1: Simulated moments for individual Kim(2000, 2002) and Busetti and Taylor tests (2004)

T	$H_1(\mathcal{K})$	$H_2(\mathcal{K})$	$H_3(\mathcal{K})$	$H_1(\mathcal{K}^*)$	$H_2(\mathcal{K}^*)$	$H_3(\mathcal{K}^*)$	\mathcal{M}^{1*}	\mathcal{M}^{2*}	\mathcal{M}^{3*}
Mean - drift case									
50	1.817	1.608	6.173	1.836	1.624	6.204	2.783	2.632	9.214
100	1.803	1.566	6.386	1.803	1.564	6.391	2.749	2.545	9.436
500	1.800	1.537	6.788	1.779	1.506	6.660	2.741	2.478	9.868
Mean - linear trend case									
50	1.415	0.906	3.785	1.412	0.908	3.796	1.992	1.334	5.297
100	1.377	0.844	3.738	1.374	0.843	3.831	1.924	1.222	5.145
500	1.362	0.815	3.718	1.353	0.822	3.839	1.886	1.177	5.115
Std. deviation - drift case									
50	1.580	2.282	5.856	1.586	2.370	6.003	1.737	2.903	6.874
100	1.548	2.178	5.841	1.526	2.106	5.728	1.669	2.644	6.590
500	1.528	2.170	6.041	1.503	1.980	5.701	1.667	2.562	6.318
Std. deviation - linear trend case									
50	0.869	0.863	2.759	0.866	0.861	2.777	0.869	1.030	3.068
100	0.814	0.687	2.451	0.798	0.685	2.447	0.794	0.781	2.618
500	0.764	0.607	2.428	0.734	0.651	2.401	0.709	0.701	2.604

Table 2: Size of Panel Tests When the Model Does Not Allow for Cross Sectional Dependence. Intercept case.

T	N	$H_1(\mathcal{K})$	$H_2(\mathcal{K})$	$H_3(\mathcal{K})$	$H_1(\mathcal{K}^*)$	$H_2(\mathcal{K}^*)$	$H_3(\mathcal{K}^*)$	\mathcal{M}^{1*}	\mathcal{M}^{2*}	\mathcal{M}^{3*}
50	25	0.075	0.074	0.078	0.069	0.061	0.058	0.056	0.061	0.063
	50	0.078	0.074	0.068	0.064	0.060	0.047	0.074	0.069	0.067
	100	0.073	0.072	0.078	0.065	0.063	0.056	0.065	0.062	0.077
100	25	0.054	0.054	0.051	0.076	0.072	0.073	0.069	0.072	0.070
	50	0.041	0.040	0.046	0.056	0.069	0.069	0.058	0.059	0.064
	100	0.051	0.060	0.064	0.053	0.065	0.055	0.049	0.051	0.051

T	N	$MH_1^d(\mathcal{K})$	$MH_2^d(\mathcal{K})$	$MH_3^d(\mathcal{K})$	$MH_1^d(\mathcal{K}^*)$	$MH_2^d(\mathcal{K}^*)$	$MH_3^d(\mathcal{K}^*)$	MM^{1*}	MM^{2*}	MM^{3*}
50	25	0.075	0.075	0.078	0.069	0.063	0.062	0.062	0.065	0.064
	50	0.078	0.078	0.069	0.065	0.061	0.047	0.075	0.070	0.067
	100	0.073	0.072	0.070	0.065	0.066	0.056	0.066	0.064	0.073
100	25	0.055	0.054	0.052	0.078	0.078	0.076	0.070	0.075	0.072
	50	0.043	0.043	0.048	0.056	0.071	0.069	0.058	0.060	0.065
	100	0.051	0.060	0.066	0.054	0.065	0.056	0.050	0.052	0.052

T	N	$MH_{1m}(\mathcal{K})$	$MH_{2m}(\mathcal{K})$	$MH_{3m}(\mathcal{K})$	$MH_{1m}(\mathcal{K}^*)$	$MH_{2m}(\mathcal{K}^*)$	$MH_{3m}(\mathcal{K}^*)$	MM_m^{1*}	MM_m^{2*}	MM_m^{3*}
50	25	0.075	0.075	0.076	0.069	0.063	0.062	0.059	0.064	0.063
	50	0.068	0.068	0.069	0.065	0.061	0.047	0.075	0.070	0.067
	100	0.073	0.072	0.070	0.065	0.065	0.056	0.065	0.064	0.072
100	25	0.055	0.054	0.051	0.077	0.073	0.072	0.060	0.074	0.071
	50	0.043	0.044	0.046	0.056	0.068	0.068	0.058	0.059	0.065
	100	0.051	0.060	0.064	0.054	0.065	0.056	0.049	0.051	0.052

Notes : Empirical sizes corresponding to a 5% nominal size.

Table 3: Power of Panel Tests When the Model Does Not Allow for Cross Sectional Dependence. $I(0) \rightarrow I(1)$. Intercept case.

τ	σ_η	T	N	$H_1(\mathcal{K})$	$H_2(\mathcal{K})$	$H_3(\mathcal{K})$	$H_1(\mathcal{K}^*)$	$H_2(\mathcal{K}^*)$	$H_3(\mathcal{K}^*)$	\mathcal{M}^{1*}	\mathcal{M}^{2*}	\mathcal{M}^{3*}
0.3	0.10	50	25	0.582	0.610	0.611	0.193	0.209	0.107	0.544	0.574	0.561
			50	0.845	0.858	0.854	0.272	0.316	0.130	0.798	0.821	0.802
			100	0.979	0.985	0.987	0.417	0.490	0.147	0.971	0.981	0.968
		100	25	0.998	0.998	0.997	0.801	0.832	0.470	0.999	0.999	0.998
			50	1.000	1.000	1.000	0.950	0.965	0.640	1.000	1.000	1.000
			100	1.000	1.000	1.000	0.997	0.997	0.806	1.000	1.000	1.000
		50	25	0.534	0.557	0.640	0.035	0.048	0.012	0.346	0.372	0.444
			50	0.819	0.818	0.902	0.030	0.042	0.007	0.588	0.609	0.669
			100	0.943	0.959	0.983	0.014	0.022	0.000	0.775	0.813	0.879
0.5	0.10	100	25	0.994	0.994	0.999	0.125	0.152	0.004	0.983	0.990	0.992
			50	1.000	1.000	1.000	0.117	0.158	0.001	0.998	0.998	1.000
			100	1.000	1.000	1.000	0.097	0.164	0.000	0.998	0.998	1.000
		50	25	0.280	0.305	0.406	0.025	0.032	0.020	0.150	0.165	0.226
			50	0.438	0.456	0.628	0.016	0.023	0.005	0.199	0.210	0.361
			100	0.639	0.662	0.866	0.008	0.011	0.003	0.317	0.349	0.554
		100	25	0.909	0.920	0.973	0.003	0.009	0.002	0.701	0.749	0.885
			50	0.989	0.990	0.999	0.001	0.003	0.000	0.888	0.921	0.984
			100	1.000	1.000	1.000	0.000	0.000	0.000	1.000	1.000	1.000
0.3	0.50	50	25	1.000	1.000	1.000	0.994	0.995	0.930	1.000	1.000	1.000
			50	1.000	1.000	1.000	1.000	1.000	0.996	1.000	1.000	1.000
			100	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		100	10	1.000	1.000	1.000	0.974	0.985	0.861	1.000	1.000	1.000
			25	1.000	1.000	1.000	1.000	1.000	0.995	1.000	1.000	1.000
			50	1.000	1.000	1.000	1.000	0.999	1.000	1.000	1.000	1.000
		100	100	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		50	25	1.000	1.000	1.000	0.636	0.685	0.051	1.000	1.000	1.000
			50	1.000	1.000	1.000	0.834	0.872	0.034	1.000	1.000	1.000
			100	1.000	1.000	1.000	0.952	0.979	0.018	1.000	1.000	1.000
0.5	0.50	100	25	1.000	1.000	1.000	0.914	0.933	0.232	1.000	1.000	1.000
			50	1.000	1.000	1.000	0.982	0.987	0.263	1.000	1.000	1.000
			100	1.000	1.000	1.000	0.999	0.999	0.244	1.000	1.000	1.000
		50	25	1.000	1.000	1.000	0.000	0.000	0.000	1.000	1.000	1.000
			50	1.000	1.000	1.000	0.000	0.000	0.000	1.000	1.000	1.000
			100	1.000	1.000	1.000	0.000	0.000	0.000	1.000	1.000	1.000
		100	25	1.000	1.000	1.000	0.000	0.000	0.000	1.000	1.000	1.000
			50	1.000	1.000	1.000	0.001	0.000	0.000	1.000	1.000	1.000
			100	1.000	1.000	1.000	0.000	0.000	0.000	1.000	1.000	1.000

Table 4: Power of Modified Panel Tests When the Model Does Not Allow for Cross Sectional Dependence. $I(0) \rightarrow I(1)$. Intercept case.

τ	σ_η	T	N	MH ₁ ^d (κ)	MH ₂ ^d (κ)	MH ₃ ^d (κ)	MH ₁ ^d (κ^*)	MH ₂ ^d (κ^*)	MH ₃ ^d (κ^*)	MM ^{1*}	MM ^{2*}	MM ^{3*}
0.3	0.10	50	25	0.588	0.612	0.615	0.197	0.214	0.108	0.557	0.578	0.574
			50	0.851	0.863	0.854	0.275	0.317	0.132	0.802	0.827	0.808
			100	0.980	0.985	0.988	0.420	0.491	0.148	0.972	0.981	0.969
			25	0.998	0.998	0.997	0.803	0.836	0.471	0.999	0.999	0.998
			50	1.000	1.000	1.000	0.951	0.965	0.643	1.000	1.000	1.000
			100	1.000	1.000	1.000	0.997	0.997	0.809	1.000	1.000	1.000
		100	25	0.542	0.560	0.647	0.038	0.048	0.013	0.356	0.382	0.451
			50	0.823	0.822	0.903	0.033	0.042	0.007	0.591	0.619	0.676
			100	0.945	0.960	0.983	0.015	0.022	0.000	0.778	0.814	0.881
			25	0.994	0.994	0.999	0.126	0.155	0.004	0.984	0.990	0.992
			50	1.000	1.000	1.000	0.118	0.159	0.001	0.999	0.999	1.000
			100	1.000	1.000	1.000	0.099	0.164	0.000	0.982	0.987	0.999
0.5	0.10	50	25	0.289	0.309	0.410	0.025	0.033	0.021	0.159	0.170	0.230
			50	0.442	0.458	0.634	0.017	0.023	0.005	0.201	0.221	0.364
			100	0.640	0.664	0.866	0.008	0.012	0.003	0.321	0.353	0.560
			25	0.912	0.920	0.973	0.003	0.009	0.002	0.708	0.751	0.889
			50	0.989	0.990	0.999	0.001	0.003	0.000	0.889	0.924	0.984
			100	1.000	1.000	1.000	0.000	0.000	0.000	0.982	0.987	0.999
		100	25	1.000	1.000	1.000	0.994	0.995	0.933	1.000	1.000	1.000
			50	1.000	1.000	1.000	1.000	1.000	0.996	1.000	1.000	1.000
			100	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
			25	1.000	1.000	1.000	1.000	1.000	0.995	1.000	1.000	1.000
			50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
			100	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.7	0.10	50	25	1.000	1.000	1.000	0.651	0.695	0.052	1.000	1.000	1.000
			50	1.000	1.000	1.000	0.835	0.876	0.034	1.000	1.000	1.000
			100	1.000	1.000	1.000	0.953	0.979	0.018	1.000	1.000	1.000
			25	1.000	1.000	1.000	0.916	0.935	0.237	1.000	1.000	1.000
			50	1.000	1.000	1.000	0.982	0.987	0.265	1.000	1.000	1.000
			100	1.000	1.000	1.000	0.999	0.999	0.247	1.000	1.000	1.000
		100	10	0.999	1.000	1.000	0.002	0.005	0.001	0.990	0.995	1.000
			25	1.000	1.000	1.000	0.000	0.000	0.000	1.000	1.000	1.000
			50	1.000	1.000	1.000	0.000	0.000	0.000	1.000	1.000	1.000
			100	1.000	1.000	1.000	0.000	0.000	0.000	1.000	1.000	1.000
			25	1.000	1.000	1.000	0.000	0.000	0.000	1.000	1.000	1.000
			50	1.000	1.000	1.000	0.001	0.001	0.000	1.000	1.000	1.000
			100	1.000	1.000	1.000	0.000	0.000	0.000	1.000	1.000	1.000

Table 5: Power of Modified min Panel Tests When the Model Does Not Allow for Cross Sectional Dependence. $I(0) \rightarrow I(1)$. Intercept case.

τ	σ_η	T	N	MH _{1m} (\mathcal{K})	MH _{2m} (\mathcal{K})	MH _{3m} (\mathcal{K})	MH _{1m} (\mathcal{K}^*)	MH _{2m} (\mathcal{K}^*)	MH _{3m} (\mathcal{K}^*)	$\mathcal{M}\mathcal{M}_m^{1*}$	$\mathcal{M}\mathcal{M}_m^{2*}$	$\mathcal{M}\mathcal{M}_m^{3*}$
0.3	0.10	50	25	0.586	0.612	0.612	0.197	0.213	0.108	0.552	0.576	0.574
			50	0.849	0.859	0.854	0.273	0.316	0.132	0.801	0.824	0.806
			100	0.979	0.985	0.988	0.419	0.491	0.147	0.972	0.981	0.969
			25	0.998	0.998	0.997	0.802	0.832	0.470	0.999	0.999	0.998
			50	1.000	1.000	1.000	0.951	0.965	0.642	1.000	1.000	1.000
			100	1.000	1.000	1.000	0.997	0.997	0.806	1.000	1.000	1.000
		100	25	0.539	0.558	0.647	0.037	0.048	0.013	0.354	0.379	0.450
			50	0.821	0.819	0.903	0.032	0.042	0.007	0.590	0.617	0.673
			100	0.998	0.998	0.997	0.802	0.832	0.470	0.999	0.999	0.998
			25	0.994	0.994	0.999	0.125	0.152	0.004	0.984	0.990	0.992
			50	1.000	1.000	1.000	0.117	0.158	0.001	0.998	0.998	1.000
			100	1.000	1.000	1.000	0.098	0.164	0.000	1.000	1.000	1.000
0.5	0.10	50	25	0.287	0.307	0.409	0.025	0.032	0.021	0.154	0.168	0.230
			50	0.441	0.456	0.633	0.017	0.023	0.005	0.200	0.220	0.363
			100	0.640	0.663	0.866	0.008	0.012	0.003	0.320	0.353	0.560
			25	0.911	0.920	0.973	0.003	0.009	0.001	0.702	0.751	0.887
			50	0.989	0.990	0.999	0.001	0.003	0.000	0.888	0.922	0.984
			100	1.000	1.000	1.000	0.098	0.164	0.000	1.000	1.000	1.000
		100	25	1.000	1.000	1.000	0.994	0.995	0.930	1.000	1.000	1.000
			50	1.000	1.000	1.000	1.000	1.000	0.996	1.000	1.000	1.000
			100	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
			25	1.000	1.000	1.000	1.000	1.000	0.995	1.000	1.000	1.000
			50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
			100	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.7	0.10	50	25	1.000	1.000	1.000	0.642	0.687	0.051	1.000	1.000	1.000
			50	1.000	1.000	1.000	0.834	0.872	0.034	1.000	1.000	1.000
			100	1.000	1.000	1.000	0.952	0.979	0.018	0.932	0.949	0.933
			25	1.000	1.000	1.000	0.915	0.933	0.233	1.000	1.000	1.000
			50	1.000	1.000	1.000	0.982	0.987	0.263	1.000	1.000	1.000
			100	1.000	1.000	1.000	0.999	0.999	0.244	1.000	1.000	1.000
		100	25	1.000	1.000	1.000	0.000	0.000	0.000	1.000	1.000	1.000
			50	1.000	1.000	1.000	0.000	0.000	0.000	1.000	1.000	1.000
			100	1.000	1.000	1.000	0.000	0.000	0.000	1.000	1.000	1.000
			25	1.000	1.000	1.000	0.000	0.000	0.000	1.000	1.000	1.000
			50	1.000	1.000	1.000	0.000	0.001	0.001	1.000	1.000	1.000
			100	1.000	1.000	1.000	0.000	0.000	0.000	1.000	1.000	1.000

Table 6: Power of Panel Tests When the Model Does Not Allow for Cross Sectional Dependence. $I(1) \rightarrow I(0)$. Intercept case.

τ	σ_η	T	N	$H_1(\mathcal{K})$	$H_2(\mathcal{K})$	$H_3(\mathcal{K})$	$H_1(\mathcal{K}^*)$	$H_2(\mathcal{K}^*)$	$H_3(\mathcal{K}^*)$	\mathcal{M}^{1*}	\mathcal{M}^{2*}	\mathcal{M}^{3*}
0.3	0.10	50	25	0.030	0.037	0.021	0.266	0.284	0.391	0.160	0.182	0.249
			50	0.021	0.036	0.012	0.427	0.428	0.593	0.206	0.218	0.355
			100	0.008	0.011	0.005	0.639	0.674	0.839	0.319	0.358	0.549
		100	25	0.002	0.003	0.001	0.891	0.913	0.968	0.682	0.736	0.875
			50	0.000	0.000	0.000	0.990	0.993	0.998	0.893	0.931	0.976
			100	0.000	0.000	0.000	1.000	1.000	1.000	0.983	0.990	1.000
		50	25	0.041	0.049	0.017	0.550	0.570	0.657	0.382	0.405	0.453
			50	0.032	0.047	0.007	0.792	0.810	0.890	0.563	0.598	0.703
			100	0.020	0.034	0.001	0.964	0.973	0.993	0.820	0.856	0.907
0.5		100	25	0.095	0.127	0.004	0.995	0.996	0.998	0.975	0.981	0.985
			50	0.095	0.134	0.000	1.000	1.000	1.000	0.998	0.999	1.000
			100	0.089	0.141	0.000	1.000	1.000	1.000	1.000	1.000	1.000
		50	25	0.220	0.247	0.120	0.623	0.639	0.636	0.593	0.619	0.599
			50	0.326	0.370	0.134	0.858	0.860	0.860	0.814	0.841	0.819
			100	0.437	0.512	0.174	0.984	0.985	0.981	0.967	0.978	0.975
		100	25	0.789	0.817	0.429	0.996	0.999	0.996	0.995	0.997	0.996
			50	0.941	0.950	0.595	1.000	1.000	1.000	1.000	1.000	1.000
			100	0.999	0.998	0.758	1.000	1.000	1.000	1.000	1.000	1.000
0.7		50	25	0.000	0.000	0.000	1.000	1.000	1.000	1.000	1.000	1.000
			50	0.000	0.000	0.000	1.000	1.000	1.000	1.000	1.000	1.000
			100	0.000	0.002	0.030	1.000	1.000	1.000	1.000	1.000	1.000
		100	25	0.000	0.001	0.000	1.000	1.000	1.000	1.000	1.000	1.000
			50	0.000	0.000	0.000	1.000	1.000	1.000	1.000	1.000	1.000
			100	0.000	0.000	0.000	1.000	1.000	1.000	1.000	1.000	1.000
		50	25	0.659	0.708	0.039	1.000	1.000	1.000	1.000	1.000	1.000
			50	0.846	0.887	0.039	1.000	1.000	1.000	1.000	1.000	1.000
			100	0.966	0.981	0.030	1.000	1.000	1.000	1.000	1.000	1.000
0.3	0.50	100	25	0.910	0.921	0.195	1.000	1.000	1.000	1.000	1.000	1.000
			50	0.095	0.133	0.000	1.000	1.000	1.000	1.000	1.000	1.000
			100	0.999	0.999	0.238	1.000	1.000	1.000	1.000	1.000	1.000
		50	25	0.999	0.999	0.937	1.000	1.000	1.000	1.000	1.000	1.000
			50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
			100	0.966	0.981	0.030	1.000	1.000	1.000	1.000	1.000	1.000
		100	25	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
			50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
			100	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 7: Power of Modified Panel Tests When the Model Does Not Allow for Cross Sectional Dependence. $I(1) \rightarrow I(0)$. Intercept case.

τ	σ_η	T	N	MH ₁ ^d (κ)	MH ₂ ^d (κ)	MH ₃ ^d (κ)	MH ₁ ^d (κ^*)	MH ₂ ^d (κ^*)	MH ₃ ^d (κ^*)	MM ^{1*}	MM ^{2*}	MM ^{3*}
0.3	0.10	50	25	0.030	0.037	0.021	0.266	0.284	0.391	0.160	0.182	0.249
			50	0.021	0.036	0.012	0.427	0.428	0.593	0.206	0.218	0.355
			100	0.008	0.011	0.005	0.639	0.674	0.839	0.319	0.358	0.549
			25	0.002	0.003	0.001	0.891	0.913	0.968	0.682	0.736	0.875
			50	0.000	0.000	0.000	0.990	0.993	0.998	0.893	0.931	0.976
			100	0.000	0.000	0.000	1.000	1.000	1.000	0.983	0.990	1.000
		100	25	0.041	0.049	0.017	0.550	0.570	0.657	0.382	0.405	0.453
			50	0.032	0.047	0.007	0.792	0.810	0.890	0.563	0.598	0.703
			100	0.020	0.034	0.001	0.964	0.973	0.993	0.820	0.856	0.907
			25	0.095	0.127	0.004	0.995	0.996	0.998	0.975	0.981	0.985
			50	0.095	0.134	0.000	1.000	1.000	1.000	0.998	0.999	1.000
			100	0.089	0.141	0.000	1.000	1.000	1.000	1.000	1.000	1.000
0.5	0.10	50	25	0.220	0.247	0.120	0.623	0.639	0.636	0.593	0.619	0.599
			50	0.326	0.370	0.134	0.858	0.860	0.860	0.814	0.841	0.819
			100	0.437	0.512	0.174	0.984	0.985	0.981	0.967	0.978	0.975
			25	0.789	0.817	0.429	0.996	0.999	0.996	0.995	0.997	0.996
			50	0.941	0.950	0.595	1.000	1.000	1.000	1.000	1.000	1.000
			100	0.999	0.998	0.758	1.000	1.000	1.000	1.000	1.000	1.000
		100	25	0.000	0.000	0.000	1.000	1.000	1.000	1.000	1.000	1.000
			50	0.000	0.000	0.000	1.000	1.000	1.000	1.000	1.000	1.000
			100	0.000	0.002	0.030	1.000	1.000	1.000	1.000	1.000	1.000
			25	0.000	0.001	0.000	1.000	1.000	1.000	1.000	1.000	1.000
			50	0.000	0.000	0.000	1.000	1.000	1.000	1.000	1.000	1.000
			100	0.000	0.000	0.000	1.000	1.000	1.000	1.000	1.000	1.000
0.7	0.10	50	25	0.659	0.708	0.039	1.000	1.000	1.000	1.000	1.000	1.000
			50	0.846	0.887	0.039	1.000	1.000	1.000	1.000	1.000	1.000
			100	0.966	0.981	0.030	1.000	1.000	1.000	1.000	1.000	1.000
			25	0.910	0.921	0.195	1.000	1.000	1.000	1.000	1.000	1.000
			50	0.095	0.133	0.000	1.000	1.000	1.000	1.000	1.000	1.000
			100	0.999	0.999	0.238	1.000	1.000	1.000	1.000	1.000	1.000
		100	25	0.999	0.999	0.937	1.000	1.000	1.000	1.000	1.000	1.000
			50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
			100	0.966	0.981	0.030	1.000	1.000	1.000	1.000	1.000	1.000
			25	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
			50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
			100	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 8: Power of Modified min Panel Tests When the Model Does Not Allow for Cross Sectional Dependence. $I(1) \rightarrow I(0)$. Intercept case.

τ	σ_η	T	N	MH _{1m} (\mathcal{K})	MH _{2m} (\mathcal{K})	MH _{3m} (\mathcal{K})	MH _{1m} (\mathcal{K}^*)	MH _{2m} (\mathcal{K}^*)	MH _{3m} (\mathcal{K}^*)	$\mathcal{M}\mathcal{M}_m^{1*}$	$\mathcal{M}\mathcal{M}_m^{2*}$	$\mathcal{M}\mathcal{M}_m^{3*}$
0.3	0.10	50	25	0.030	0.037	0.021	0.266	0.284	0.391	0.160	0.182	0.249
			50	0.021	0.036	0.012	0.427	0.428	0.593	0.206	0.218	0.355
			100	0.008	0.011	0.005	0.639	0.674	0.839	0.319	0.358	0.549
			25	0.002	0.003	0.001	0.891	0.913	0.968	0.682	0.736	0.875
			50	0.000	0.000	0.000	0.990	0.993	0.998	0.893	0.931	0.976
			100	0.000	0.000	0.000	1.000	1.000	1.000	0.983	0.990	1.000
		100	25	0.041	0.049	0.017	0.550	0.570	0.657	0.382	0.405	0.453
			50	0.032	0.047	0.007	0.792	0.810	0.890	0.563	0.598	0.703
			100	0.020	0.034	0.001	0.964	0.973	0.993	0.820	0.856	0.907
			25	0.095	0.127	0.004	0.995	0.996	0.998	0.975	0.981	0.986
			50	0.095	0.134	0.000	1.000	1.000	1.000	0.998	0.999	1.000
			100	0.089	0.141	0.000	1.000	1.000	1.000	1.000	1.000	1.000
0.5	0.10	50	25	0.220	0.247	0.120	0.623	0.639	0.636	0.593	0.619	0.599
			50	0.326	0.370	0.134	0.858	0.860	0.860	0.814	0.841	0.819
			100	0.437	0.512	0.174	0.984	0.985	0.981	0.967	0.978	0.975
			25	0.789	0.817	0.429	0.996	0.999	0.996	0.995	0.997	0.996
			50	0.941	0.950	0.595	1.000	1.000	1.000	1.000	1.000	1.000
			100	0.999	0.998	0.758	1.000	1.000	1.000	1.000	1.000	1.000
		100	25	0.000	0.000	0.000	1.000	1.000	1.000	1.000	1.000	1.000
			50	0.000	0.000	0.000	1.000	1.000	1.000	1.000	1.000	1.000
			100	0.000	0.000	0.000	1.000	1.000	1.000	1.000	1.000	1.000
			25	0.000	0.001	0.000	1.000	1.000	1.000	1.000	1.000	1.000
			50	0.000	0.000	0.000	1.000	1.000	1.000	1.000	1.000	1.000
			100	0.000	0.000	0.000	1.000	1.000	1.000	1.000	1.000	1.000
0.7	0.10	50	25	0.659	0.708	0.039	1.000	1.000	1.000	1.000	1.000	1.000
			50	0.846	0.887	0.039	1.000	1.000	1.000	1.000	1.000	1.000
			100	0.966	0.981	0.030	1.000	1.000	1.000	1.000	1.000	1.000
			25	0.910	0.921	0.195	1.000	1.000	1.000	1.000	1.000	1.000
			50	0.984	0.991	0.238	1.000	1.000	1.000	1.000	1.000	1.000
			100	0.999	0.999	0.238	1.000	1.000	1.000	1.000	1.000	1.000
		100	25	0.999	0.999	0.940	1.000	1.000	1.000	1.000	1.000	1.000
			50	1.000	1.000	1.000	0.992	1.000	1.000	1.000	1.000	1.000
			100	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
			25	1.000	1.000	0.994	1.000	1.000	1.000	1.000	1.000	1.000
			50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
			100	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 9: Size of Panel Tests When the Model Allows for Low and Strong Cross Sectional Dependence. Intercept case.

A/Low										
T	N	$H_1(\mathcal{K})$	$H_2(\mathcal{K})$	$H_3(\mathcal{K})$	$H_1(\mathcal{K}^*)$	$H_2(\mathcal{K}^*)$	$H_3(\mathcal{K}^*)$	\mathcal{M}^{1*}	\mathcal{M}^{2*}	\mathcal{M}^{3*}
50	25	0.181	0.183	0.185	0.081	0.062	0.076	0.165	0.156	0.165
	50	0.199	0.190	0.194	0.083	0.068	0.076	0.210	0.204	0.200
	100	0.230	0.223	0.230	0.105	0.093	0.104	0.236	0.229	0.228
100	10	0.179	0.176	0.174	0.070	0.075	0.070	0.176	0.173	0.177
	25	0.179	0.173	0.181	0.083	0.091	0.085	0.185	0.180	0.188
	50	0.229	0.216	0.227	0.104	0.090	0.103	0.231	0.221	0.235
T	N	$MH_1^d(\mathcal{K})$	$MH_2^d(\mathcal{K})$	$MH_3^d(\mathcal{K})$	$MH_1^d(\mathcal{K}^*)$	$MH_2^d(\mathcal{K}^*)$	$MH_3^d(\mathcal{K}^*)$	MM^{1*}	MM^{2*}	MM^{3*}
50	25	0.185	0.186	0.185	0.082	0.063	0.076	0.170	0.157	0.166
	50	0.200	0.191	0.197	0.085	0.069	0.077	0.210	0.207	0.202
	100	0.235	0.223	0.233	0.108	0.099	0.106	0.236	0.231	0.227
100	25	0.179	0.174	0.184	0.095	0.083	0.085	0.187	0.180	0.189
	50	0.230	0.217	0.228	0.104	0.090	0.106	0.231	0.223	0.238
	100	0.236	0.226	0.240	0.112	0.103	0.112	0.241	0.239	0.247
T	N	$MH_{1m}(\mathcal{K})$	$MH_{2m}(\mathcal{K})$	$MH_{3m}(\mathcal{K})$	$MH_{1m}(\mathcal{K}^*)$	$MH_{2m}(\mathcal{K}^*)$	$MH_{3m}(\mathcal{K}^*)$	MM_m^{1*}	MM_m^{2*}	MM_m^{3*}
50	25	0.182	0.185	0.185	0.082	0.063	0.076	0.169	0.157	0.166
	50	0.200	0.191	0.196	0.085	0.069	0.077	0.210	0.207	0.202
	100	0.235	0.223	0.232	0.108	0.098	0.104	0.236	0.231	0.227
100	25	0.179	0.174	0.183	0.095	0.083	0.085	0.185	0.180	0.188
	50	0.229	0.217	0.227	0.104	0.090	0.105	0.231	0.222	0.236
	100	0.245	0.233	0.241	0.118	0.101	0.104	0.236	0.231	0.227
B/High										
T	N	$H_1(\mathcal{K})$	$H_2(\mathcal{K})$	$H_3(\mathcal{K})$	$H_1(\mathcal{K}^*)$	$H_2(\mathcal{K}^*)$	$H_3(\mathcal{K}^*)$	\mathcal{M}^{1*}	\mathcal{M}^{2*}	\mathcal{M}^{3*}
50	25	0.419	0.409	0.381	0.230	0.205	0.200	0.402	0.381	0.391
	50	0.477	0.458	0.433	0.255	0.246	0.215	0.425	0.406	0.425
	100	0.542	0.523	0.521	0.321	0.331	0.310	0.405	0.401	0.434
100	25	0.365	0.360	0.334	0.220	0.210	0.180	0.355	0.331	0.320
	50	0.431	0.423	0.401	0.255	0.246	0.215	0.429	0.416	0.409
	100	0.512	0.502	0.521	0.321	0.331	0.310	0.472	0.481	0.478
T	N	$MH_1^d(\mathcal{K})$	$MH_2^d(\mathcal{K})$	$MH_3^d(\mathcal{K})$	$MH_1^d(\mathcal{K}^*)$	$MH_2^d(\mathcal{K}^*)$	$MH_3^d(\mathcal{K}^*)$	MM^{1*}	MM^{2*}	MM^{3*}
50	25	0.420	0.410	0.383	0.230	0.208	0.203	0.409	0.402	0.384
	50	0.478	0.458	0.434	0.257	0.246	0.215	0.443	0.406	0.427
	100	0.541	0.523	0.519	0.323	0.331	0.310	0.432	0.401	0.436
100	25	0.367	0.362	0.334	0.221	0.212	0.180	0.357	0.333	0.321
	50	0.428	0.414	0.380	0.208	0.251	0.232	0.399	0.391	0.372
	100	0.539	0.510	0.512	0.302	0.298	0.310	0.412	0.342	0.412
T	N	$MH_{1m}(\mathcal{K})$	$MH_{2m}(\mathcal{K})$	$MH_{3m}(\mathcal{K})$	$MH_{1m}(\mathcal{K}^*)$	$MH_{2m}(\mathcal{K}^*)$	$MH_{3m}(\mathcal{K}^*)$	MM_m^{1*}	MM_m^{2*}	MM_m^{3*}
50	25	0.420	0.410	0.383	0.230	0.205	0.203	0.407	0.382	0.392
	50	0.477	0.459	0.434	0.256	0.215	0.246	0.442	0.427	0.406
	100	0.540	0.522	0.519	0.323	0.324	0.334	0.432	0.418	0.423
100	25	0.365	0.360	0.334	0.221	0.211	0.180	0.356	0.332	0.320
	50	0.428	0.414	0.379	0.251	0.230	0.207	0.399	0.391	0.371
	100	0.539	0.513	0.509	0.320	0.331	0.310	0.410	0.401	0.410

Notes : Empirical sizes corresponding to a 5% nominal size.

Table 10: Size of DeFactored Panel Tests. Intercept case

T	N	$H_1(\mathcal{K})$	$H_2(\mathcal{K})$	$H_3(\mathcal{K})$	$H_1(\mathcal{K}^*)$	$H_2(\mathcal{K}^*)$	$H_3(\mathcal{K}^*)$	\mathcal{M}^{1*}	\mathcal{M}^{2*}	\mathcal{M}^{3*}
50	25	0.066	0.067	0.066	0.065	0.063	0.063	0.059	0.056	0.061
	50	0.082	0.076	0.077	0.061	0.063	0.050	0.065	0.070	0.070
	100	0.080	0.077	0.080	0.059	0.058	0.059	0.071	0.068	0.074
100	25	0.064	0.062	0.063	0.070	0.072	0.068	0.066	0.065	0.065
	50	0.055	0.057	0.048	0.069	0.066	0.068	0.062	0.065	0.059
	100	0.057	0.055	0.501	0.068	0.067	0.057	0.063	0.062	0.053

T	N	$MH_1^d(\mathcal{K})$	$MH_2^d(\mathcal{K})$	$MH_3^d(\mathcal{K})$	$MH_1^d(\mathcal{K}^*)$	$MH_2^d(\mathcal{K}^*)$	$MH_3^d(\mathcal{K}^*)$	$M\mathcal{M}^{1*}$	$M\mathcal{M}^{2*}$	$M\mathcal{M}^{3*}$
50	25	0.069	0.067	0.070	0.067	0.066	0.064	0.062	0.058	0.064
	50	0.074	0.076	0.079	0.062	0.065	0.050	0.075	0.075	0.073
	100	0.080	0.077	0.081	0.060	0.058	0.059	0.073	0.069	0.074
100	25	0.064	0.062	0.063	0.069	0.072	0.068	0.065	0.065	0.064
	50	0.055	0.057	0.048	0.068	0.066	0.067	0.059	0.064	0.059
	100	0.057	0.054	0.051	0.067	0.065	0.057	0.061	0.061	0.053

T	N	$MH_{1m}(\mathcal{K})$	$MH_{2m}(\mathcal{K})$	$MH_{3m}(\mathcal{K})$	$MH_{1m}(\mathcal{K}^*)$	$MH_{2m}(\mathcal{K}^*)$	$MH_{3m}(\mathcal{K}^*)$	$M\mathcal{M}_m^{1*}$	$M\mathcal{M}_m^{2*}$	$M\mathcal{M}_m^{3*}$
50	25	0.069	0.067	0.069	0.067	0.065	0.063	0.061	0.057	0.063
	50	0.072	0.076	0.079	0.061	0.064	0.050	0.075	0.075	0.073
	100	0.080	0.077	0.081	0.059	0.058	0.059	0.061	0.069	0.074
	25	0.064	0.064	0.064	0.070	0.072	0.069	0.067	0.066	0.066
	50	0.056	0.058	0.049	0.069	0.066	0.068	0.063	0.065	0.059
	100	0.057	0.055	0.051	0.068	0.067	0.057	0.063	0.062	0.053

Notes :Empirical sizes corresponding to a 5% nominal size.

Table 11: Power of Defactored Panel Tests. $I(0) \rightarrow I(1)$. Intercept case.

τ	σ_η	T	N	$H_1(\mathcal{K})$	$H_2(\mathcal{K})$	$H_3(\mathcal{K})$	$H_1(\mathcal{K}^*)$	$H_2(\mathcal{K}^*)$	$H_3(\mathcal{K}^*)$	\mathcal{M}^1	\mathcal{M}^{2*}	\mathcal{M}^{3*}
0.3	0.10	50	25	0.400	0.408	0.405	0.161	0.171	0.088	0.350	0.364	0.335
			50	0.500	0.521	0.503	0.191	0.193	0.093	0.456	0.442	0.418
			100	0.684	0.672	0.657	0.255	0.264	0.103	0.624	0.625	0.595
		100	25	0.834	0.821	0.798	0.454	0.485	0.201	0.821	0.811	0.737
			50	0.877	0.829	0.763	0.325	0.287	0.051	0.810	0.843	0.814
			100	0.912	0.893	0.865	0.574	0.291	0.034	0.900	0.893	0.861
0.5	50	25	0.467	0.480	0.545	0.049	0.056	0.029	0.310	0.321	0.369	
		50	0.688	0.661	0.740	0.053	0.057	0.041	0.134	0.140	0.154	
		100	0.886	0.901	0.947	0.021	0.027	0.008	0.674	0.721	0.796	
	100	25	0.899	0.900	0.908	0.079	0.105	0.006	0.850	0.859	0.860	
		50	0.555	0.506	0.494	0.082	0.078	0.013	0.437	0.404	0.319	
		100	0.625	0.513	0.399	0.112	0.081	0.019	0.436	0.344	0.130	
0.7	50	25	0.297	0.315	0.419	0.027	0.026	0.019	0.153	0.166	0.249	
		50	0.438	0.436	0.434	0.025	0.030	0.031	0.199	0.210	0.298	
		100	0.593	0.633	0.817	0.007	0.008	0.004	0.275	0.317	0.484	
	100	25	0.849	0.864	0.943	0.006	0.008	0.002	0.635	0.688	0.822	
		50	0.966	0.966	0.969	0.006	0.006	0.003	0.834	0.873	0.946	
		100	0.998	0.978	0.999	0.053	0.052	0.072	0.919	0.956	0.997	
0.3	0.50	50	25	0.678	0.625	0.583	0.064	0.068	0.039	0.394	0.372	0.283
			50	0.761	0.656	0.632	0.027	0.031	0.017	0.436	0.412	0.391
			100	0.876	0.777	0.748	0.007	0.007	0.004	0.611	0.575	0.560
		100	25	0.997	0.992	0.984	0.076	0.082	0.042	0.940	0.925	0.854
			50	0.999	0.997	0.997	0.026	0.030	0.014	0.991	0.988	0.966
			100	1.000	1.000	1.000	0.001	0.001	0.002	0.999	0.999	0.997
0.5	50	25	0.609	0.594	0.455	0.025	0.021	0.014	0.698	0.679	0.644	
		50	0.996	0.997	0.996	0.030	0.042	0.033	0.993	0.993	0.993	
		100	0.999	0.999	0.999	0.018	0.016	0.027	0.998	0.999	0.999	
	100	25	0.947	0.920	0.949	0.009	0.014	0.000	0.692	0.665	0.608	
		50	0.995	0.985	0.994	0.001	0.001	0.000	0.837	0.805	0.747	
		100	1.000	1.000	1.000	0.000	0.000	0.000	0.944	0.919	0.897	
0.7	50	25	0.174	0.144	0.112	0.059	0.047	0.055	0.312	0.295	0.263	
		50	0.242	0.218	0.198	0.247	0.174	0.319	0.335	0.329	0.311	
		100	0.347	0.284	0.260	0.429	0.372	0.481	0.503	0.559	0.570	
	100	25	0.528	0.502	0.464	0.010	0.009	0.009	0.439	0.426	0.374	
		50	0.717	0.677	0.533	0.033	0.030	0.033	0.665	0.649	0.614	
		100	0.947	0.928	0.882	0.063	0.053	0.099	0.853	0.834	0.806	

Table 12: Power of Defactored Modified Panel Tests. $I(0) \rightarrow I(1)$. Intercept case.

τ	σ_η	T	N	$\text{MH}_1^{\text{d}}(\kappa)$	$\text{MH}_2^{\text{d}}(\kappa)$	$\text{MH}_3^{\text{d}}(\kappa)$	$\text{MH}_1^{\text{d}}(\kappa^*)$	$\text{MH}_2^{\text{d}}(\kappa^*)$	$\text{MH}_3^{\text{d}}(\kappa^*)$	MM^{1*}	MM^{2*}	MM^{3*}
0.3	0.10	50	25	0.409	0.412	0.410	0.165	0.174	0.089	0.357	0.372	0.343
			50	0.551	0.568	0.559	0.189	0.200	0.092	0.473	0.498	0.465
			100	0.684	0.673	0.661	0.258	0.264	0.104	0.626	0.625	0.596
		100	25	0.834	0.821	0.798	0.453	0.485	0.199	0.821	0.810	0.737
			50	0.877	0.829	0.814	0.324	0.287	0.305	0.882	0.856	0.834
			100	0.969	0.927	0.901	0.290	0.246	0.034	0.979	0.958	0.921
0.5	50	25	0.477	0.486	0.546	0.051	0.057	0.029	0.317	0.327	0.377	
		50	0.688	0.735	0.810	0.054	0.058	0.043	0.498	0.510	0.623	
		100	0.888	0.902	0.947	0.021	0.027	0.008	0.682	0.712	0.796	
	100	25	0.898	0.900	0.907	0.079	0.105	0.005	0.848	0.858	0.859	
		50	0.939	0.943	0.951	0.082	0.078	0.013	0.900	0.910	0.916	
		100	0.993	0.999	0.999	0.112	0.081	0.019	0.967	0.987	0.978	
0.7	50	25	0.300	0.320	0.421	0.027	0.027	0.019	0.159	0.173	0.262	
		50	0.453	0.487	0.578	0.025	0.030	0.032	0.212	0.245	0.324	
		100	0.597	0.637	0.819	0.008	0.008	0.004	0.284	0.319	0.485	
	100	25	0.849	0.864	0.943	0.006	0.008	0.002	0.634	0.687	0.820	
		50	0.966	0.966	0.969	0.006	0.006	0.003	0.834	0.873	0.946	
		100	0.999	0.999	0.999	0.053	0.052	0.072	0.910	0.923	0.991	
0.3	0.50	50	25	0.689	0.626	0.584	0.064	0.070	0.039	0.407	0.383	0.289
			50	0.762	0.659	0.635	0.028	0.032	0.018	0.589	0.579	0.465
			100	0.877	0.781	0.750	0.007	0.007	0.004	0.724	0.710	0.634
		100	25	0.997	0.992	0.984	0.076	0.082	0.042	0.940	0.924	0.853
			50	0.999	0.997	0.997	0.026	0.030	0.014	0.991	0.987	0.963
			100	1.000	1.000	1.000	0.001	0.002	0.001	0.999	0.997	0.997
0.5	50	25	0.375	0.308	0.294	0.025	0.023	0.012	0.187	0.157	0.124	
		50	0.510	0.489	0.467	0.027	0.021	0.015	0.298	0.246	0.210	
		100	0.789	0.720	0.698	0.018	0.016	0.028	0.410	0.378	0.324	
	100	25	0.947	0.919	0.948	0.009	0.014	0.000	0.691	0.664	0.605	
		50	0.995	0.985	0.994	0.001	0.001	0.000	0.837	0.802	0.746	
		100	0.999	0.999	1.000	0.000	0.000	0.000	0.942	0.999	0.997	
0.7	50	25	0.176	0.145	0.114	0.063	0.051	0.057	0.116	0.095	0.067	
		50	0.245	0.212	0.189	0.250	0.179	0.322	0.176	0.149	0.123	
		100	0.349	0.286	0.246	0.430	0.375	0.481	0.207	0.196	0.178	
	100	25	0.528	0.501	0.464	0.010	0.009	0.009	0.439	0.426	0.374	
		50	0.712	0.700	0.678	0.033	0.030	0.031	0.598	0.587	0.534	
		100	0.887	0.854	0.819	0.063	0.053	0.098	0.810	0.778	0.730	

Table 13: Power of Defactored Modified Min Panel Tests. $I(0) \rightarrow I(1)$. Intercept case.

τ	σ_η	T	N	MH _{1m} (\mathcal{K})	MH _{2m} (\mathcal{K})	MH _{3m} (\mathcal{K})	MH _{1m} (\mathcal{K}^*)	MH _{2m} (\mathcal{K}^*)	MH _{3m} (\mathcal{K}^*)	M \mathcal{M}_m^{1*}	M \mathcal{M}_m^{2*}	M \mathcal{M}_m^{3*}
0.3	0.10	50	25	0.403	0.410	0.407	0.162	0.173	0.089	0.354	0.368	0.340
			50	0.548	0.563	0.552	0.187	0.200	0.092	0.471	0.491	0.462
			100	0.684	0.672	0.661	0.258	0.264	0.103	0.626	0.625	0.595
		100	25	0.836	0.821	0.799	0.458	0.488	0.201	0.824	0.811	0.737
			50	0.877	0.830	0.764	0.325	0.287	0.051	0.884	0.858	0.833
			100	0.968	0.929	0.902	0.295	0.246	0.034	0.971	0.967	0.921
0.5	50	25	0.474	0.482	0.546	0.050	0.057	0.029	0.315	0.325	0.375	
		50	0.683	0.731	0.798	0.053	0.057	0.042	0.496	0.508	0.621	
		100	0.887	0.901	0.947	0.021	0.027	0.008	0.678	0.712	0.796	
	100	25	0.900	0.900	0.909	0.080	0.107	0.006	0.850	0.859	0.860	
		50	0.941	0.943	0.953	0.082	0.079	0.013	0.900	0.910	0.916	
		100	0.996	0.999	0.999	0.113	0.082	0.019	0.967	0.987	0.978	
0.7	50	25	0.297	0.317	0.421	0.027	0.027	0.019	0.156	0.168	0.259	
		50	0.450	0.483	0.578	0.025	0.030	0.032	0.210	0.234	0.321	
		100	0.597	0.636	0.819	0.007	0.008	0.004	0.282	0.318	0.484	
	100	25	0.849	0.865	0.943	0.006	0.008	0.002	0.640	0.693	0.823	
		50	0.966	0.966	0.969	0.006	0.006	0.003	0.834	0.873	0.947	
		100	0.998	0.999	0.999	0.054	0.052	0.073	0.912	0.922	0.990	
0.3	0.50	50	25	0.682	0.625	0.583	0.064	0.069	0.039	0.404	0.378	0.287
			50	0.762	0.657	0.635	0.028	0.032	0.017	0.585	0.572	0.463
			100	0.876	0.779	0.748	0.007	0.007	0.004	0.785	0.708	0.635
		100	25	0.997	0.993	0.984	0.076	0.082	0.042	0.941	0.928	0.859
			50	0.999	0.997	0.997	0.027	0.030	0.015	0.991	0.988	0.968
			100	1.000	1.000	1.000	0.001	0.002	0.001	0.999	0.999	0.997
0.5	50	25	0.369	0.305	0.293	0.025	0.023	0.012	0.186	0.156	0.123	
		50	0.500	0.482	0.466	0.026	0.021	0.014	0.297	0.246	0.209	
		100	0.778	0.712	0.697	0.018	0.016	0.027	0.410	0.298	0.323	
	100	25	0.997	0.993	0.984	0.076	0.082	0.042	0.941	0.928	0.856	
		50	0.995	0.985	0.994	0.001	0.001	0.000	0.841	0.807	0.750	
		100	0.999	0.999	1.000	0.000	0.000	0.000	0.945	0.919	0.898	
0.7	50	25	0.175	0.144	0.114	0.062	0.050	0.057	0.116	0.095	0.067	
		50	0.244	0.211	0.200	0.248	0.178	0.322	0.036	0.176	0.123	
		100	0.347	0.285	0.265	0.429	0.374	0.481	0.207	0.210	0.179	
	100	25	0.529	0.503	0.464	0.010	0.010	0.009	0.441	0.426	0.375	
		50	0.711	0.702	0.678	0.033	0.031	0.034	0.601	0.587	0.374	
		100	0.875	0.857	0.818	0.064	0.053	0.101	0.810	0.778	0.730	

Table 14: Power of Defactored Panel Tests. $I(1) \rightarrow I(0)$. Intercept case.

τ	σ_η	T	N	$H_1(\mathcal{K})$	$H_2(\mathcal{K})$	$H_3(\mathcal{K})$	$H_1(\mathcal{K}^*)$	$H_2(\mathcal{K}^*)$	$H_3(\mathcal{K}^*)$	\mathcal{M}^1	\mathcal{M}^{2*}	\mathcal{M}^{3*}
0.3	0.10	50	25	0.039	0.044	0.030	0.252	0.260	0.366	0.154	0.165	0.202
			50	0.039	0.037	0.031	0.398	0.403	0.550	0.184	0.199	0.265
			100	0.009	0.014	0.009	0.577	0.596	0.757	0.244	0.284	0.457
		100	25	0.003	0.009	0.001	0.851	0.875	0.954	0.616	0.681	0.817
			50	0.000	0.004	0.002	0.953	0.955	0.964	0.839	0.883	0.939
			100	0.070	0.055	0.079	0.993	0.995	0.999	0.934	0.971	0.989
		50	25	0.045	0.057	0.026	0.422	0.431	0.540	0.300	0.315	0.368
			50	0.062	0.071	0.049	0.632	0.657	0.687	0.523	0.543	0.589
			100	0.030	0.043	0.006	0.878	0.888	0.938	0.682	0.725	0.812
0.5		100	25	0.058	0.087	0.009	0.921	0.923	0.924	0.868	0.884	0.888
			50	0.067	0.062	0.010	0.965	0.967	0.970	0.921	0.943	0.951
			100	0.101	0.081	0.014	0.978	0.999	0.998	0.987	0.999	0.999
		50	25	0.138	0.155	0.095	0.386	0.382	0.384	0.346	0.365	0.357
			50	0.120	0.117	0.087	0.510	0.508	0.509	0.472	0.483	0.476
			100	0.264	0.304	0.114	0.667	0.647	0.634	0.638	0.633	0.600
		100	25	0.444	0.452	0.203	0.815	0.808	0.768	0.793	0.773	0.704
			50	0.297	0.274	0.053	0.869	0.827	0.743	0.910	0.890	0.838
			100	0.280	0.222	0.022	0.930	0.910	0.887	0.987	0.975	0.929
0.3	0.50	50	25	0.057	0.052	0.073	0.151	0.126	0.105	0.089	0.083	0.050
			50	0.205	0.205	0.339	0.226	0.212	0.146	0.146	0.139	0.112
			100	0.409	0.341	0.448	0.375	0.301	0.170	0.254	0.180	0.171
		100	25	0.010	0.012	0.010	0.523	0.500	0.467	0.427	0.415	0.366
			50	0.023	0.021	0.024	0.718	0.693	0.640	0.578	0.561	0.470
			100	0.055	0.036	0.087	0.836	0.825	0.798	0.741	0.721	0.653
		50	25	0.034	0.035	0.015	0.357	0.313	0.294	0.215	0.195	0.129
			50	0.021	0.021	0.015	0.756	0.678	0.633	0.374	0.328	0.223
			100	0.027	0.024	0.037	0.881	0.849	0.819	0.532	0.471	0.378
0.5		100	25	0.024	0.028	0.003	0.955	0.920	0.948	0.706	0.685	0.642
			50	0.002	0.003	0.000	0.995	0.991	0.997	0.836	0.805	0.745
			100	0.000	0.000	0.000	1.000	1.000	1.000	0.953	0.936	0.915
		50	25	0.064	0.070	0.037	0.698	0.614	0.583	0.394	0.363	0.281
			50	0.022	0.018	0.007	0.747	0.774	0.719	0.525	0.491	0.361
			100	0.007	0.007	0.006	0.848	0.737	0.750	0.674	0.561	0.444
		100	25	0.071	0.082	0.034	0.997	0.993	0.989	0.940	0.926	0.850
			50	0.117	0.017	0.021	0.998	0.998	0.997	0.992	0.988	0.969
			100	0.003	0.003	0.003	1.000	1.000	1.000	0.998	0.997	0.994

Table 15: Power of Defactored Modified Panel Tests. $I(1) \rightarrow I(0)$. Intercept case.

τ	σ_η	T	N	$\text{MH}_1^{\text{d}}(\kappa)$	$\text{MH}_2^{\text{d}}(\kappa)$	$\text{MH}_3^{\text{d}}(\kappa)$	$\text{MH}_1^{\text{d}}(\kappa^*)$	$\text{MH}_2^{\text{d}}(\kappa^*)$	$\text{MH}_3^{\text{d}}(\kappa^*)$	MM^{1*}	MM^{2*}	MM^{3*}
0.3	0.10	50	25	0.040	0.045	0.032	0.260	0.269	0.373	0.159	0.168	0.210
			50	0.039	0.039	0.032	0.401	0.414	0.567	0.192	0.134	0.179
			100	0.010	0.015	0.009	0.583	0.599	0.760	0.250	0.286	0.460
		100	25	0.003	0.010	0.001	0.852	0.875	0.954	0.622	0.686	0.823
			50	0.000	0.004	0.002	0.953	0.955	0.964	0.839	0.884	0.939
			100	0.071	0.056	0.080	0.992	0.995	0.999	0.935	0.972	0.989
		50	25	0.048	0.058	0.026	0.428	0.438	0.547	0.311	0.324	0.375
			50	0.065	0.071	0.051	0.639	0.663	0.701	0.542	0.561	0.610
			100	0.030	0.043	0.006	0.880	0.889	0.939	0.688	0.729	0.815
0.5	100	25	0.059	0.088	0.009	0.923	0.923	0.915	0.885	0.886	0.890	
		50	0.068	0.063	0.010	0.967	0.967	0.971	0.991	0.959	0.953	
		100	0.102	0.082	0.014	0.610	0.518	0.389	0.443	0.999	0.999	
	50	25	0.142	0.156	0.097	0.391	0.388	0.390	0.355	0.373	0.362	
		50	0.120	0.118	0.087	0.515	0.510	0.511	0.478	0.490	0.478	
		100	0.269	0.307	0.114	0.669	0.647	0.636	0.640	0.633	0.603	
	100	25	0.449	0.457	0.207	0.819	0.811	0.769	0.795	0.777	0.705	
		50	0.298	0.277	0.054	0.869	0.828	0.799	0.797	0.790	0.780	
		100	0.280	0.222	0.022	0.930	0.911	0.876	0.878	0.901	0.864	
0.3	0.50	50	25	0.059	0.056	0.074	0.155	0.129	0.105	0.091	0.086	0.055
			50	0.264	0.208	0.339	0.229	0.214	0.146	0.151	0.142	0.103
			100	0.409	0.343	0.449	0.378	0.302	0.171	0.256	0.184	0.173
		100	25	0.010	0.013	0.010	0.527	0.502	0.468	0.428	0.416	0.367
			50	0.023	0.022	0.024	0.722	0.695	0.641	0.579	0.562	0.470
			100	0.057	0.037	0.088	0.037	0.025	0.002	0.742	0.722	0.652
		50	25	0.039	0.038	0.016	0.359	0.317	0.296	0.224	0.199	0.132
			50	0.028	0.024	0.019	0.759	0.695	0.631	0.379	0.325	0.228
			100	0.029	0.024	0.038	0.891	0.854	0.819	0.532	0.473	0.378
100	25	0.024	0.028	0.003	0.957	0.922	0.951	0.716	0.687	0.646		
	50	0.002	0.003	0.000	0.995	0.991	0.997	0.842	0.810	0.746		
	100	0.000	0.000	0.000	1.000	1.000	1.000	0.999	0.999	0.999		
0.7		50	25	0.064	0.072	0.037	0.702	0.625	0.590	0.407	0.373	0.287
			50	0.021	0.021	0.015	0.758	0.682	0.637	0.545	0.513	0.365
			100	0.007	0.008	0.006	0.850	0.737	0.723	0.678	0.643	0.445
		100	25	0.072	0.082	0.035	0.997	0.993	0.989	0.944	0.927	0.855
			50	0.017	0.021	0.009	0.998	0.998	0.997	0.992	0.989	0.971
			100	0.003	0.003	0.003	1.000	1.000	1.000	0.998	0.997	0.995

Table 16: Power of Defactored Modified Min Panel Tests. $I(1) \rightarrow I(0)$. Intercept case.

τ	σ_η	T	N	MH _{1m} (\mathcal{K})	MH _{2m} (\mathcal{K})	MH _{3m} (\mathcal{K})	MH _{1m} (\mathcal{K}^*)	MH _{2m} (\mathcal{K}^*)	MH _{3m} (\mathcal{K}^*)	MA _m ^{1*}	MA _m ^{2*}	MA _m ^{3*}
0.3	0.10	50	25	0.040	0.045	0.032	0.256	0.263	0.371	0.157	0.167	0.209
			50	0.039	0.038	0.032	0.401	0.407	0.558	0.189	0.201	0.271
			100	0.010	0.015	0.009	0.581	0.597	0.760	0.249	0.284	0.460
		100	25	0.003	0.009	0.001	0.852	0.875	0.954	0.617	0.682	0.819
			50	0.000	0.004	0.002	0.953	0.955	0.964	0.839	0.883	0.939
			100	0.070	0.055	0.080	0.994	0.996	0.999	0.934	0.971	0.989
		50	25	0.047	0.057	0.026	0.424	0.432	0.545	0.307	0.321	0.373
			50	0.063	0.071	0.051	0.634	0.659	0.681	0.529	0.551	0.591
			100	0.030	0.043	0.006	0.879	0.889	0.938	0.685	0.727	0.814
0.5	100	25	0.058	0.087	0.009	0.923	0.923	0.924	0.870	0.884	0.889	
		50	0.068	0.063	0.010	0.972	0.972	0.970	0.920	0.942	0.949	
		100	0.101	0.082	0.014	0.991	0.991	0.970	0.986	0.999	0.999	
	50	25	0.140	0.156	0.097	0.389	0.386	0.388	0.354	0.369	0.361	
		50	0.120	0.118	0.087	0.515	0.519	0.512	0.479	0.489	0.481	
		100	0.269	0.305	0.114	0.669	0.647	0.635	0.640	0.633	0.602	
	100	25	0.444	0.452	0.203	0.815	0.811	0.769	0.795	0.776	0.704	
		50	0.297	0.274	0.053	0.869	0.827	0.789	0.910	0.890	0.838	
		100	0.280	0.222	0.022	0.930	0.910	0.876	0.987	0.975	0.929	
0.3	0.50	50	25	0.059	0.054	0.074	0.154	0.129	0.105	0.090	0.086	0.055
			50	0.263	0.208	0.339	0.229	0.218	0.146	0.146	0.137	0.116
			100	0.409	0.342	0.449	0.378	0.302	0.171	0.255	0.184	0.172
		100	25	0.010	0.012	0.010	0.523	0.501	0.467	0.428	0.415	0.366
			50	0.023	0.022	0.024	0.718	0.695	0.640	0.579	0.562	0.480
			100	0.055	0.036	0.088	0.836	0.821	0.799	0.744	0.722	0.663
		50	25	0.037	0.037	0.016	0.359	0.316	0.295	0.220	0.196	0.131
			50	0.028	0.024	0.019	0.756	0.679	0.631	0.381	0.329	0.225
			100	0.029	0.024	0.037	0.882	0.850	0.829	0.551	0.471	0.379
0.5	0.7	100	25	0.024	0.028	0.003	0.956	0.920	0.949	0.709	0.686	0.642
			50	0.002	0.003	0.000	0.995	0.991	0.997	0.839	0.806	0.745
			100	0.000	0.000	0.000	1.000	1.000	1.000	0.953	0.936	0.915
		50	25	0.064	0.071	0.037	0.699	0.621	0.584	0.402	0.368	0.283
			50	0.021	0.021	0.015	0.758	0.679	0.633	0.535	0.499	0.374
			100	0.007	0.008	0.006	0.850	0.737	0.722	0.689	0.573	0.459
		100	25	0.072	0.082	0.035	0.997	0.993	0.989	0.940	0.926	0.851
			50	0.017	0.021	0.009	0.998	0.998	0.997	0.992	0.989	0.969
			100	0.003	0.003	0.003	1.000	1.000	1.000	0.998	0.997	0.995

Table 17: Inflation rate 1970.1-2007.3

No cross-sectional dependence adjusted tests					
Test Statistics	test – value (p – values)	Test Statistics	test – value (p – values)	Test Statistics	test – value (p – values)
$H_1(\mathcal{K})$	1.664 (0.048)	$H_1(\mathcal{K}^*)$	408.373 (0.000)	\mathcal{M}^{1*}	352.873 (0.000)
$H_2(\mathcal{K})$	-0.876 (0.809)	$H_2(\mathcal{K}^*)$	246.559 (0.000)	\mathcal{M}^{2*}	222.889 (0.000)
$H_3(\mathcal{K})$	1.415 (0.079)	$H_3(\mathcal{K}^*)$	549.627 (0.000)	\mathcal{M}^{3*}	436.048 (0.000)
$MH_1^d(\mathcal{K})$	1.731 (0.023)	$MH_1^d(\mathcal{K}^*)$	413.061 (0.000)	MM^{1*}	357.636 (0.000)
$MH_2^d(\mathcal{K})$	-0.844 (0.801)	$MH_2^d(\mathcal{K}^*)$	248.708 (0.000)	MM^{2*}	225.168 (0.000)
$MH_3^d(\mathcal{K})$	1.474 (0.070)	$MH_3^d(\mathcal{K}^*)$	557.550 (0.000)	MM^{3*}	443.112 (0.000)
$MH_{1m}(\mathcal{K})$	1.667 (0.048)	$MH_{1m}(\mathcal{K}^*)$	408.617 (0.000)	MM_m^{1*}	353.187 (0.000)
$MH_{2m}(\mathcal{K})$	-0.874 (0.809)	$MH_{2m}(\mathcal{K}^*)$	246.677 (0.000)	MM_m^{2*}	223.052 (0.000)
$MH_{3m}(\mathcal{K})$	1.418 (0.078)	$MH_{3m}(\mathcal{K}^*)$	550.015 (0.000)	MM_m^{3*}	436.187 (0.000)
Break Date : 1977.3					
Cross-correlation dependence adjusted tests					
Test Statistics	test – value (p – values)	Test Statistics	test – value (p – values)	Test Statistics	test – value (p – values)
$H_1(\mathcal{K})$	-0.846 (0.801)	$H_1(\mathcal{K}^*)$	55.396 (0.000)	\mathcal{M}^{1*}	46.288 (0.000)
$H_2(\mathcal{K})$	-3.103 (0.999)	$H_2(\mathcal{K}^*)$	39.024 (0.000)	\mathcal{M}^{2*}	33.092 (0.000)
$H_3(\mathcal{K})$	-0.946 (0.846)	$H_3(\mathcal{K}^*)$	70.435 (0.000)	\mathcal{M}^{3*}	54.639 (0.000)
$MH_1^d(\mathcal{K})$	-0.829 (0.797)	$MH_1^d(\mathcal{K}^*)$	55.649 (0.000)	MM^{1*}	46.547 (0.000)
$MH_2^d(\mathcal{K})$	-3.097 (0.999)	$MH_2^d(\mathcal{K}^*)$	39.161 (0.000)	MM^{2*}	33.236 (0.000)
$MH_3^d(\mathcal{K})$	-0.933 (0.824)	$MH_3^d(\mathcal{K}^*)$	70.830 (0.000)	MM^{3*}	54.994 (0.000)
$MH_{1m}(\mathcal{K})$	-0.844 (0.801)	$MH_{1m}(\mathcal{K}^*)$	55.419 (0.000)	MM_m^{1*}	46.321 (0.000)
$MH_{2m}(\mathcal{K})$	-3.102 (0.999)	$MH_{2m}(\mathcal{K}^*)$	33.113 (0.000)	MM_m^{2*}	33.113 (0.000)
$MH_{3m}(\mathcal{K})$	-0.945 (0.828)	$MH_{3m}(\mathcal{K}^*)$	70.468 (0.000)	MM_m^{3*}	54.682 (0.000)
Break Date : 1980.4					